Q#1
a. The annual interest rate is 10%.

\[ PV = \frac{FV(T=2)}{(1+k)^2} \]

\[ 1,000 = \frac{1,210}{(1+k)^2} \Rightarrow (1+k)^2 = \frac{1,210}{1,000} \Rightarrow k = \sqrt{\frac{1,210}{1,000}} - 1 = 0.1 = 10\% \]

b. The annual interest rate is approximately 4.88%.

The only difference is that now \( T = 4 \), therefore

\[ 1,000 = \frac{1,210}{(1+k)^4} \Rightarrow (1+k)^4 = \frac{1,210}{1,000} \Rightarrow k = \sqrt[4]{\frac{1,210}{1,000}} - 1 \approx 0.0488 = 4.88\% \]

Q#2
The discount rate is approximately 16.67%.

The time-line representation of the second payment option is

\[ \begin{array}{c|c|c|c|c|c|c|c|c} 0 & 1 & 2 & \ldots & t \end{array} \]

where \( CF_0 = 200,000 \) and \( CF_t = 350,000 \).

The two payment options are equivalent if their present values are the same, i.e.

\[ 500,000 = 200,000 + \frac{350,000}{1+k} \Rightarrow k = \frac{350,000}{(500,000 - 200,000)} - 1 \approx 0.1667 = 16.67\% \]

Q#3
a. The present value of the dividend stream is $50.

This is a growing perpetuity and its general time-line representation is

\[ \begin{array}{c|c|c|c|c} 0 & 1 & 2 & \ldots & t \end{array} \]

We know that \( CF = 1 \), \( g = 3\% \), and \( k = 5\% \), therefore

\[ PV = \frac{1}{k-g} = \frac{50}{0.05 - 0.03} = \frac{1}{0.02} = 50 \quad (\text{note that } k = 5\% > 3\% = g) \]

b. The present value of the dividend stream is $20.

One should expect a reduction in the present value \( PV \) since there is an inverse relationship between the \( PV \) and the discount rate \( (k) \): the \( PV \) gets smaller when the \( k \) increases.

\[ PV = \frac{1}{k-g} = \frac{20}{0.08 - 0.03} = \frac{1}{0.05} = 20 \quad (\text{note that } k = 8\% > 3\% = g) \]
Q#4
a. If the investment horizon is 1 year, the effective annual interest rate offered by bank A is the highest (10% > 9.925% > 9.417%).

\[ FV(\$1, \ k = 10\% \text{ compounded annually, } T = 1 \text{ year}) = (1 + 0.1) = \$1.1 \]

\[ FV(\$1, \ k = 9.5\% \text{ compounded monthly, } T = 1 \text{ year}) = \left(1 + \frac{0.095}{12}\right)^{12} \approx \$1.09925 \]

\[ FV(\$1, \ k = 9\% \text{ compounded continuously, } T = 1 \text{ year}) = e^{0.09} \approx \$1.09417 \]

b. If the investment horizon is 10 years, the effective 10 year return offered by bank A is still the highest one (159.374% > 157.606% > 145.960%).

\[ FV(\$1, \ k = 10\% \text{ compounded annually, } T = 10 \text{ years}) = (1 + 0.1)^{10} \approx \$2.59374 \]

\[ FV(\$1, \ k = 9.5\% \text{ compounded monthly, } T = 10 \text{ years}) = \left(1 + \frac{0.095}{12}\right)^{120} \approx 2.57606 \]

\[ FV(\$1, \ k = 9\% \text{ compounded continuously, } T = 10 \text{ years}) = (e^{0.09})^{10} \approx \$2.45960 \]

Note that the additional compounding over 10 years, which works the same way for all the effective annual interest rates, can not reverse the initial ordering of the banks resulting from our calculation of the effective annual interest rate.

Q#5
Each annual installment is approximately $13,297.

This is an annuity and its general time-line representation is

\[ \begin{aligned} &\text{CF} &\text{CF}(1 + g) &\ldots &\text{CF}(1 + g)^{(T-1)} \\
&0 &1 &2 &\ldots &T=30 \end{aligned} \]

We know that \( PV = $165,000, \quad g = 0\%, \quad \text{and} \quad k = 7\%, \quad \text{therefore} \)

\[ PV = \frac{\text{CF}}{k - g} \left[ 1 - \left(\frac{1 + g}{1 + k}\right)^T \right] \quad \text{(note that \( k = 7\% > 0\% = g \))} \]

\[ $165,000 = \frac{\text{CF}}{0.07} \left[ 1 - \left(\frac{1}{1 + 0.07}\right)^{30} \right] \quad \Rightarrow \quad \text{CF} = \frac{$165,000}{0.07} \left[ 1 - \left(\frac{1}{1 + 0.07}\right)^{30} \right] \approx $13,297 \]

Q#6
The current price of the bond is $1,098.4.

This is an annuity plus an additional CF (the principal) on T = 6. The time-line representation of the CF stream is

\[ \begin{aligned} &\text{CF} &\text{CF} &\ldots &\text{CF} + \text{principal} \\
&0 &1 &2 &\ldots &T=6 \end{aligned} \]

We know that the principal is $1,000, \( CF = $1,000 \times 0.08 = $80, \quad \text{and} \quad k = 6\%, \quad \text{therefore} \]
\[
PV = \frac{CF}{k} \left[ 1 - \left( \frac{1}{1+k} \right)^T \right] + \frac{\text{principal}}{(1+k)^T}
\]

\[
PV = \frac{\$80}{0.06} \left[ 1 - \left( \frac{1}{1+0.06} \right)^6 \right] + \frac{\$1,000}{(1+0.06)^6} \approx \$1,098.4
\]

Note that we use the par value (or the principal) and the bond coupon information to determine the stream of cash flows. The yield on other bonds with “similar” risk characteristics is a measure of the risk adjusted rate of return. The par value is not the price of the bond, and it doesn’t have to be the price even when the bond is issued.

**Q#7**

**The current price of the stock is approximately $397.06.**

We have a combination of two CF streams, one is a growing annuity and the other is a growing perpetuity. Though one could calculate the PV as the sum of two PVs of the two CF streams it is easier to proceed as follows.

The time-line representation of the CF stream is

\[
\begin{array}{cccccccc}
\text{0} & \text{1} & \text{2} & \text{3} & \text{4} & \text{5} & \text{...} \\
\hline
\end{array}
\]

and the PV is

\[
PV = \frac{CF_1}{(1+k)} + \frac{CF(1+g_1)}{(1+k)^2} \left( \frac{k-g}{(1+k)^2} \right) \quad \text{(note that } k = 8\% > 5\% = g_1)\]

\[
PV = \frac{10}{(1+0.08)} + \frac{10(1+0.15)}{(1+0.08)^2} + \frac{10(1+0.15)}{(1+0.08)^2} \approx \$397.06
\]

Note that there is more than one way to calculate the PV of this CF stream. You could use \( t = 3 \) rather than \( t = 2 \) as the zero point for the growing perpetuity calculation. If you are careful the present value will obviously remain the same.

\[
PV = \frac{10}{(1+0.08)} + \frac{10(1+0.15)}{(1+0.08)^2} + \frac{10(1+0.15)^2}{(1+0.08)^2} + \frac{10(1+0.15)^2(1+0.05)}{(1+0.08)^3} \approx \$397.06
\]

**Q#8**

a. Each deposit should be approximately $855.6.

Note that we will use both, the PV of annuity formula and the relationship between the PV and the FV\( (T = 5) \).

The time-line representation of the annuity is

\[
\begin{array}{cccccccc}
\text{0} & \text{1} & \text{2} & \text{3} & \text{4} & \text{T} = 5 \\
\hline
\end{array}
\]

The stated annual rate (compounded quarterly) is 16\%, therefore the effective annual interest rate is
\[ 1 + k_{\text{eff}} = \left( 1 + \frac{k_{\text{stated}}}{m} \right)^m = \left( 1 + \frac{0.16}{4} \right)^4 = 1.16985856 \implies k_{\text{eff}} = 16.985856\% \]

If we use the relationship between the FV(T=5) and the PV we get

\[ PV = \frac{FV}{(1 + k)^T} = \frac{\$6,000}{(1.16985856)^5} \approx \$2,738.3, \]

and if we plug that number into the calculation of the PV of an annuity we get

\[ PV = \frac{CF}{k} \left[ 1 - \left( \frac{1}{1+k} \right)^T \right] \]

\[ \$2,738.3 \approx \frac{CF}{0.16985856} \left[ 1 - \left( \frac{1}{1.16985856} \right)^4 \right] \implies CF \approx \$855.6 \]

b. Each deposit should be approximately $997.9.

Note that now we simply have 4 rather than 5 equal installments (i.e. the 5th year CF equals zero). Using the PV formula for of the new annuity we get

\[ \$2,738.3 \approx \frac{CF}{0.16985856} \left[ 1 - \left( \frac{1}{1.16985856} \right)^4 \right] \implies CF \approx \$997.9 \]

c. Each deposit should be approximately $731.4.

Note that now we have five equal installments at the beginning of each year (rather than at the end of the year as in a). Using the PV formula of the new CF stream we get

\[ \$2,738.3 \approx CF + \frac{CF}{0.16985856} \left[ 1 - \left( \frac{1}{1.16985856} \right)^4 \right] \implies CF \approx \$731.4 \]

A shorter calculation relies on the fact that we move each CF in the CF stream described in (a) one year back in time. Therefore, the answer could be obtained by using the PV and FV relationship as follows

\[ PV = \frac{FV}{1+k_{\text{eff}}} \implies CF_c = \frac{CF_a}{1+k_{\text{eff}}} \approx \frac{\$855.6}{1+0.16985856} \approx \$731.4 \]