Part I – Open Questions

1. Rita’s utility of final wealth is defined by \( U(W) = \sqrt{W} \). She is broke but one lucky day she found a lottery ticket with two possible outcomes: $196 with probability 0.25 and $100 with probability 0.75.

1a. What is the expected utility of Rita’s final wealth? (4 points)

1b. What is the lowest price that Rita will accept for her lottery ticket?

1c. My utility of final wealth is also defined by \( U(W) = \sqrt{W} \), but I have $1,000 in my pocket. What is my utility of my final wealth? (4 points)

1d. I offer to buy the lottery ticket from Rita for $122. If I buy the ticket, what is my expected utility of final wealth? (4 points)

(Note that if I had $1,000 and paid $122 for the ticket, then my final wealth should be $(1,000-122) + \text{the payoff of the lottery ticket}).

1e. Comparing your answers to c (without the ticket) and d (with the ticket), did buying the lottery ticket for $122 increase my welfare? (4 points)

\[
\begin{align*}
1a. \quad & EU(W) = 0.25 \cdot \sqrt{196} + 0.75 \cdot \sqrt{100} = 11 \text{ welfare units} \\
1b. \quad & UC = CE = 11 = \Rightarrow CE = 11^2 = 121 \\
1c. \quad & U(1,000) = \sqrt{1,000} = 31.62278 \text{ welfare units} \\
1d. \quad & EU(W) = 0.25 \sqrt{1,196-122} + 0.75 \sqrt{1,100-122} \\
& = 31.64773 \text{ welfare units} \\
1e. \quad & Yes, \quad U(1,000) = 31.62278 < 31.64773 = EU(W) \quad \text{without the lottery ticket} \\
& \uparrow \quad \text{with the lottery ticket.}
\end{align*}
\]
2. The RiskAreUs family can invest only in one of the following risky assets (with normally-distributed returns):

<table>
<thead>
<tr>
<th></th>
<th>Expected Return - ( \mu )</th>
<th>Std of the Return - ( \sigma )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8%</td>
<td>20%</td>
<td>1.1</td>
</tr>
<tr>
<td>B</td>
<td>10%</td>
<td>15%</td>
<td>1.7</td>
</tr>
<tr>
<td>C</td>
<td>5%</td>
<td>20%</td>
<td>0.2</td>
</tr>
</tbody>
</table>

2a. Mr. RiskAreUs is risk-neutral, which investment does he prefer? – Explain (no more than 10 words) (5 points).

2b. Mrs. RiskAreUs is risk-averse, which investment does she prefer? – Explain (no more than 20 words) (5 points).

2c. Junior points out that something must be wrong with the data, since \( \sigma_A > \sigma_B \) but at the same time \( \beta_A < \beta_B \). Is Junior’s statement correct? – Explain (no more than 20 words and two sketches. I think that a good explanation in this case should include a sketch of assets A and B in the \( \mu-\sigma \) and \( \mu-\beta \) planes) (5 points).

2a. Mr. RiskAreUs prefers to invest in asset \( B \) because, \ldots (max 10 words).

\[ \max \{ \mu_A, \mu_B, \mu_C \} = \mu_B \]

**Risk-neutral investors don't care about the risk.**

2b. Mrs. RiskAreUs prefers to invest in asset \( B \) because, \ldots (max 20 words).

\( B \) is the only \( M-V \) efficient investment:

(i) \( \mu_B > \mu_A \) and \( \sigma_B \leq \sigma_A \)

(ii) \( \mu_B > \mu_C \) and \( \sigma_B \leq \sigma_C \).
In the context of the CAPM investors can diversify and they are compensated only for the systematic risk.

Counter example (to Junior's argument):
3. Suppose there are two risky assets, A and B, with the following distribution of returns

<table>
<thead>
<tr>
<th>The state of the economy</th>
<th>Probability</th>
<th>( R_A )</th>
<th>( R_B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.2</td>
<td>(-2%)</td>
<td>(-5%)</td>
</tr>
<tr>
<td>Normal</td>
<td>0.5</td>
<td>0%</td>
<td>10%</td>
</tr>
<tr>
<td>Expansion</td>
<td>0.3</td>
<td>20%</td>
<td>15%</td>
</tr>
</tbody>
</table>

3a. Calculate the vector of expected returns. (4 points)

3b. Calculate the variance-covariance matrix. (4 points)

3c. Find the global minimum-variance portfolio. (4 points)

3d. Calculate the expected return and the Std of the return of the global minimum-variance portfolio. (4 points)

3e. Calculate the expected return and the Std of the return of the portfolio \( p = \{w_A=0.7, \ w_B=0.3\} \). Is this portfolio Mean-Variance efficient? (4 points)
3d. \[ \mu_{\text{min}} = 0.115 \times 5.6\% + 0.885 \times 8.5\% = 8.2\% \]

\[ \sigma_{\text{min}} = \sqrt{(0.115)^2 \times 89.44 + (0.885)^2 \times 50.25 + 2 \times 0.115 \times 0.885 \times 44.4} \]

\[ = 7\% \]

3e. \[ P = \begin{bmatrix} W_A = 0.7 \\ W_B = 0.3 \end{bmatrix} \]

\[ \Rightarrow \mu_P = 6.47\% \]

\[ \sigma_P = 8.19\% \]

\[ \Rightarrow P \text{ is not an efficient portfolio} \]

\[ \mu_P = 6.47\% < 8.2\% = \mu_{\text{min}} \]
Part II – Multiple Choice Questions

4. The minimum variance frontier I is located to the left and to the north of the minimum variance frontier II in the $\mu$-$\sigma$ plane:

Which of the following statements is correct? (9 points)

a. The minimum variance frontier I may represent portfolios with more assets than the portfolios on frontier II.

b. The minimum variance frontier I may represent portfolios of risky assets with lower correlations than the assets in the portfolios on frontier II.

c. The minimum variance frontier I may represent portfolios of assets constructed without a short sales constraint, while on frontier II we will find the portfolios constructed with a short sales constraint.

d. Statements (a), (b) and (c) are correct.

e. None of the above.

We get a "better" frontier whenever we add assets; reduce correlations; remove restrictions.
5. The expected return of the market portfolio is $\mu_m=20\%$ and the return on the risk-free asset is $r_f=4\%$. What can you say about the composition of the efficient portfolio (on the CML) that has an expected return of $\mu_p=36\%$? (9 points)
   a. We need data about the risk in the market to answer this question.
   b. We need data about the beta of the market portfolio to answer this question.
   c. $200\%$ is invested in the market portfolio and $(-100\%)$ in the risk-free asset.
   d. $20\%$ is invested in the market portfolio and $80\%$ in the risk-free asset.
   e. Such a portfolio is not feasible.

$$\mu_p = 36\% = \alpha \mu_m + (1-\alpha) r_f = \alpha 20\% + (1-\alpha) 4\%,$$
$$\Rightarrow \alpha = 200\% = 2.$$

6. There is one risk-free and only two risky assets in the economy with the following parameters:

$$\mu_a = 10\% \quad \sigma_a = 5\% \quad \rho_{ab} = (-0.5)$$
$$\mu_b = 15\% \quad \sigma_b = 10\% \quad r_f = 5\%$$

What is the proportion of asset a in the market portfolio? (9 points)
   a. 0.11
   b. 0.42
   c. 0.67
   d. 0.80
   e. None of the above.

$$\begin{align*}
W_a \times 5^2 + W_b \times 5 \times 10 \times (-0.5) &= 10 - 5 \\
W_a \times 5 \times 10 \times (-0.5) + W_b \times 10^2 &= 15 - 5
\end{align*}$$

$$\Rightarrow \begin{cases} W_a = 40 \\
W_b = 20 \\
Z_a = 0.667 \\
Z_b = 0.333
\end{cases}$$
7. The expected return of the market portfolio is \( \mu_m = 12\% \), the Std of the return on that portfolio is \( \sigma_m = 18\% \) and the return of the risk-free asset is \( r_f = 6\% \). There are two risky assets in this market with the following parameters:

<table>
<thead>
<tr>
<th>Asset</th>
<th>( \mu - \text{The (market) Expected Return} )</th>
<th>( \sigma - \text{The Std of the Return} )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15%</td>
<td>22%</td>
<td>1.5</td>
</tr>
<tr>
<td>B</td>
<td>10%</td>
<td>20%</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Assuming that the CAPM model should hold, which of the following statements is correct? (9 points)

- a. The market is in equilibrium (CAPM).
- b. B is overpriced (by the market relative to the CAPM), since it’s expected return is lower than the expected return of the market portfolio and it’s Std of return is higher than the Std of return of the market portfolio.
- c. In equilibrium (CAPM), we expect asset A to have a lower price than its current market price.
- d. In equilibrium (CAPM), we expect asset B to have a higher price than its current market price.
- e. None of the above.

8. Which of the following statements is correct? (9 points)

- a. The weak form of market efficiency says that all information in past prices is reflected in current prices.
- b. The semi-strong form of market efficiency implies that security prices reflect all available public information.
- c. The strong form of market efficiency is consistent with managers being consistently able to do better than the S&P market index (a proxy for the market portfolio) if they invest in a portfolio with a beta above 1.
- d. Only statements (a) and (b) are correct.
- e. Statements (a), (b) and (c) are correct.