Part I – Open Questions

1. The current price of stock ABC is $25.

1a. Write down the possible payoffs of a long position in a European put option on ABC stock, which expires in one year and has an exercise price of $20.

<table>
<thead>
<tr>
<th>$S_1$</th>
<th>$P_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$20$</td>
</tr>
<tr>
<td>$5$</td>
<td>$15$</td>
</tr>
<tr>
<td>$10$</td>
<td>$10$</td>
</tr>
<tr>
<td>$15$</td>
<td>$5$</td>
</tr>
<tr>
<td>$20$</td>
<td>$0$</td>
</tr>
<tr>
<td>$25$</td>
<td>$0$</td>
</tr>
<tr>
<td>$30$</td>
<td>$0$</td>
</tr>
<tr>
<td>$35$</td>
<td>$0$</td>
</tr>
<tr>
<td>$40$</td>
<td>$0$</td>
</tr>
<tr>
<td>$45$</td>
<td>$0$</td>
</tr>
<tr>
<td>$50$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

What is the formula of the payoff above?

$$P_T = \max\{X - S_T, 0\}$$

1b. Write down the possible payoffs of each of the following instruments separately, and of the portfolio of all three:

(i) A long position in a European put option on ABC stock, which expires in one year and has an exercise price of $20;

(ii) A long position in a European call option on ABC stock, which expires in one year and has an exercise price of $30;

(iii) A short position in a zero coupon bond with face value of $5 and maturity in one year.
1c. Sketch the payoff diagram for the portfolio (only the portfolio, not the individual instruments).

<table>
<thead>
<tr>
<th>$S_1$</th>
<th>Long put</th>
<th>Long call</th>
<th>Short bond</th>
<th>Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20$</td>
<td>$0$</td>
<td>$0$</td>
<td>$-5$</td>
<td>$10$</td>
</tr>
<tr>
<td>$15$</td>
<td>$0$</td>
<td>$0$</td>
<td>$-5$</td>
<td>$5$</td>
</tr>
<tr>
<td>$10$</td>
<td>$0$</td>
<td>$0$</td>
<td>$-5$</td>
<td>$0$</td>
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<td>$5$</td>
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<td>$-5$</td>
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<td>$0$</td>
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<td>$0$</td>
<td>$-5$</td>
<td>$-5$</td>
</tr>
<tr>
<td>$35$</td>
<td>$0$</td>
<td>$5$</td>
<td>$-5$</td>
<td>$0$</td>
</tr>
<tr>
<td>$40$</td>
<td>$0$</td>
<td>$10$</td>
<td>$-5$</td>
<td>$5$</td>
</tr>
<tr>
<td>$45$</td>
<td>$0$</td>
<td>$15$</td>
<td>$-5$</td>
<td>$10$</td>
</tr>
<tr>
<td>$50$</td>
<td>$0$</td>
<td>$20$</td>
<td>$-5$</td>
<td>$15$</td>
</tr>
</tbody>
</table>
1d. Sketch the payoff diagram for a short position in the portfolio (assuming the portfolio described above is the long position).

![Payoff Diagram]

1e. If you believe that the price of ABC stock will **increase by $25** (to $50) next year, will you invest in the long portfolio position? **Yes** / **No**

If you believe that the price of ABC stock will **decrease by $15** (to $10), will you invest in the long position? **Yes** / **No**

If you believe that the price of ABC stock will **not change** (stay $25) next year, will you invest in the long position? **Yes** / **No**

Is the possible **gain** from a short position in the portfolio bounded? **Yes** / **No**

Is the possible **loss** from a short position in the portfolio bounded? **Yes** / **No**

(No explanations please, just choose one of the two options.)
2. A European put option on the stock of XYZ, with an exercise price equal to \( X \) and expiration \( T \) periods from now is traded on the exchange. The price of the option has both upper and lower bounds, \( LB < p < UB \).

2a. The upper bound of the put option price is \( UB = \frac{PV(X)}{1 + r} \).

The lower bound of the put option price is \( LB = \max\left\{ PV(X) - S, 0 \right\} \).

2b. XYZ stock is traded for $20. A European put option on XYZ, with an exercise price equal to $23 and expiration 6 months from now is traded for $1.40. Is there an opportunity to make arbitrage profit if the monthly risk-free rate is 1%?

(Yes) No.

Proof (If there is no arbitrage - show that you can not make arbitrage profits, if there is arbitrage show that you can. In either case, describe the cash-flows in each point in time and each state, and explain in one sentence why the result represents no-arbitrage or an arbitrage profit.)

\[
\text{UB: } \$1.40 = p < PV(X) = \frac{\$23}{(1.01)^6} = \$21.667 \checkmark \\
\text{LB: } \$1.40 = p < \max\left\{ PV(X) - S, 0 \right\} = \max\left\{ 21.667 - 20, 0 \right\} X = \$1.667
\]

=> There is an opportunity to make arbitrage profits

Since \( p = 1.40 < 1.667 = PV(X) - S \)

\( PV(X) - S - p = 1.667 - 1.40 = 0.267 > 0 \)

Short bond, long stock, long put.
2c. There is another European option on XYZ stock, with an exercise price equal to $24 and expiration 6 months from now. This put option is traded for $1.20. How will you make an arbitrage profit using the two put options?

- Long/Short the put option with an exercise price of $24;
- Long/Short the put option with an exercise price of $23.

\[
\begin{align*}
\text{Restriction:} & \quad \frac{P(x=24)}{P(x=23)} > 1 \\
\text{Data:} & \quad P(x=23) - P(x=24) = 1.4 - 1.2 > 0
\end{align*}
\]
3. USB stock currently trades for $82 per share. Each year, there are two possible outcomes. The stock price will either increase by 12% or decrease by 10%. The risk-free interest rate is 5% per annum. You work for an investment bank that has to write a European put option on USB stock which expires in two years and has an exercise (strike) price of $80.

3a. Use binomial trees to describe the price process of USB stock, the risk free bond and the put option on USB stock.

3b. Calculate the no-arbitrage price of the European put option.

3c. What is the price of an American put option with the same characteristics (assume you will be able to exercise the American option at time $t=0$ or $t=1$ or $t=2$)?

3d. When do you expect the buyer to exercise the American option (which date or 2; which state or states: 0, $u_d$, $uu$, $ud$, $dd$)?

\[\text{Put option } X = 80\]

\[
P_u = \max\{X - 102.861, 0\} = 0
\]

\[
P_{0u} = \max\{X - 82.656, 0\} = 0
\]

\[
P_{00} = \max\{X - 66.42, 0\} = 13.58
\]
3b)\[ q_v \leq 0 \quad q_d \leq 1 \]
\[
\begin{align*}
91.84 q_v + 73.8 q_d &= 82 \\
1.05 q_v + 1.05 q_d &= 1
\end{align*}
\implies \quad q_v = 0.64935 \\
q_d = 0.30065
\]

3c) \( P_v = 0 \) (no calculations needed)
\[
P_d = 0 \times q_v + 13.58 \times q_d = 4.11515
\]
\[
P_{EU} = 0 \times q_v + 4.11515 q_d = 1.24702
\]

3d) \( P_v = 0 \) (again, no calculations needed)
\[
P_d = \max \left( 0 \times q_v + 13.58 \times q_d, \ 80 - 73.8 \right) = \max \left( 4.12, 6.2 \right)
\]
\[
= \$6.2 
\implies \text{it is optimal to exercise at this state}
\]
\[
P_{AH} = \max \left( 0 \times q_v + 6.2 \times q_d, \ 80 - 82 \right) = \max \left( 4.12, 1.8788 \right)
\]
\[
= \$1.8788 \implies \text{it is optimal to wait one period}
\]

\[1.8788 = P_{AH} < 0.1358 \quad 1.247 = P_{EU} < 0.1358 \]

early exercise
state \( d \) time \( = 1 \)
4. The current price of Bethlehem Steel stock is $123, the variance of the continuously compounded annual returns of the stock is 0.25 and the continuously compounded risk-free rate is 4% per annum.

4a. Use the Black-Scholes formula to find the value of a call option on Bethlehem Steel stock. The exercise price is $115 and the time to expiration is 48 days (assume 365 days a year).

4b. What should be the price of a put option on Bethlehem Steel stock, with an exercise price of $115 and 48 days to expiration?

4a. \[ S = 123, \quad \sigma = \sqrt{0.25} = 0.5, \quad r = 0.04, \quad T = \frac{48}{365} = 0.1315 \text{ years} \]

\[ d_1 = \frac{\ln\left(\frac{123}{115}\right) + 0.04 \times 0.1315}{0.5 \times 10.1315^{1/2}} = 0.49058 \]

\[ d_2 = 0.49058 - 0.5 \times 10.1315^{1/2} = 0.30926 \]

\[ N(d_1) \approx N(0.49) = 0.6879, \quad N(d_2) \approx N(0.31) = 0.6217 \]

\[ C = 123 \times 0.6879 - 115 \times e^{-0.04 \times 0.1315} \times 0.6217 \approx 13.49 \]

\[ P = C - [S \times e^{-rT}] = 13.49 - [123 \times 115 \times e^{-0.04 \times 0.1315}] \]

\[ P_{\text{Put-Call Parity}} = 4.89 \]

4b. \[ 6.66 \times e^{-0.04 \times 0.1315} = 6.66 \times 0.9998 = 6.66 \]
4c. If the market price of the put option described in part b is $6 and if the Black-Scholes perfect-market assumptions hold, what strategy will result in an arbitrage profit?

$6 = P > C - [S - x e^{-rT}] = 4.89$

$P - C + S - x e^{-rT} = 6 - 4.89 = 1.11$

**Long/Short** one Bethlehem Steel stock;

Buy / Write one call option on Bethlehem Steel stock, with an exercise price of $115 and 48 days to expiration;

Buy / Write one put option on Bethlehem Steel stock, with an exercise price of $115 and 48 days to expiration;

Long / Short one risk-free zero-coupon bond, which pays a continuously compounded annual interest rate of 4% and pays the face-value of $115 in 48 days.

4d. If you invest in the arbitrage portfolio described in part c, your cash-flows will be:

Time: $t=0$, $cf = 1.11 = 6 - 4.89$

Time: $t = 48$ days (expiration), state: $0 < S_T < 115$, $cf = 0$;

Time: $t = 48$ days (expiration), state: $115 < S_T < \infty$, $cf = 0$.

4e. I am not sure about my initial estimate of the stock return volatility (0.25). If you compare the Black-Scholes put option price of part b, $4.89$ to the market price of the put option in part c ($6), what can you say about the implied volatility?

The stock return volatility, implied by the market price of $6 (for the put option described in part b), is **Higher** / Lower than 0.25.
## Part II – True / False Questions

<table>
<thead>
<tr>
<th>Q</th>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>5a</td>
<td>A call option gives its owner the right to buy stock at a fixed exercise price.</td>
<td>True / False</td>
</tr>
<tr>
<td>5b</td>
<td>If you write a put option, you acquire the right to sell stock at a fixed exercise price.</td>
<td>True / False</td>
</tr>
<tr>
<td>5c</td>
<td>When a call option expires and the stock price is above the exercise price, the option is worth the stock price plus the exercise price.</td>
<td>True / False</td>
</tr>
<tr>
<td>5d</td>
<td>The value of an American call option increases if the return volatility or the time-to-expiration increase.</td>
<td>True / False</td>
</tr>
<tr>
<td>5e</td>
<td>The value of an American put option decreases if the return volatility or the time-to-expiration increase.</td>
<td>True / False</td>
</tr>
<tr>
<td>5f</td>
<td>An investor who has sold (written) a call option will be happy to see the stock price fall sharply.</td>
<td>True / False</td>
</tr>
<tr>
<td>5g</td>
<td>An investor who owns a put option will be happy to see the stock price fall sharply.</td>
<td>True / False</td>
</tr>
<tr>
<td>5h</td>
<td>An American option is at least as valuable as a European option (ceteris paribus – all else equal).</td>
<td>True / False</td>
</tr>
<tr>
<td>5i</td>
<td>In the binomial tree model, we expect the put option price to be high if there is a high probability that the stock price will decrease (ceteris paribus).</td>
<td>True / False</td>
</tr>
<tr>
<td>5j</td>
<td>In the binomial tree model, we expect the call option price to be high if we divide the time to expiration into more sub-periods (ceteris paribus).</td>
<td>True / False</td>
</tr>
</tbody>
</table>