CHAPTER 22: FUTURES MARKETS

PROBLEM SETS

1. There is little hedging or speculative demand for cement futures, since cement prices are fairly stable and predictable. The trading activity necessary to support the futures market would not materialize.

2. The ability to buy on margin is one advantage of futures. Another is the ease with which one can alter one’s holdings of the asset. This is especially important if one is dealing in commodities, for which the futures market is far more liquid than the spot market.

3. Short selling results in an immediate cash inflow, whereas the short futures position does not:

<table>
<thead>
<tr>
<th>Action</th>
<th>Initial CF</th>
<th>Final CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short Sale</td>
<td>+P₀</td>
<td>−Pₜ</td>
</tr>
<tr>
<td>Short Futures</td>
<td>0</td>
<td>F₀ − Pₜ</td>
</tr>
</tbody>
</table>

4. a. False. For any given level of the stock index, the futures price will be lower when the dividend yield is higher. This follows from spot-futures parity:

\[ F₀ = S₀ (1 + r_f − d)^T \]

b. False. The parity relationship tells us that the futures price is determined by the stock price, the interest rate, and the dividend yield; it is not a function of beta.

c. True. The short futures position will profit when the market falls. This is a negative beta position.

5. The futures price is the agreed-upon price for deferred delivery of the asset. If that price is fair, then the value of the agreement ought to be zero; that is, the contract will be a zero-NPV agreement for each trader.

6. Because long positions equal short positions, futures trading must entail a “canceling out” of bets on the asset. Moreover, no cash is exchanged at the inception of futures trading. Thus, there should be minimal impact on the spot market for the asset, and futures trading should not be expected to reduce capital available for other uses.
7. **a.** The closing futures price for the March contract was 1,477.20, which has a dollar value of:

$$250 \times 1,477.20 = 369,300$$

Therefore, the required margin deposit is: $36,930

**b.** The futures price increases by: 1,500.00 – 1,477.20 = 22.80

The credit to your margin account would be: 22.80 \times 250 = 5,700

This is a percent gain of: 5,700/36,930 = 0.1543 = 15.43%

Note that the futures price itself increased by only 1.543%.

**c.** Following the reasoning in part (b), any change in \( F \) is magnified by a ratio of \((1/margin~requirement)\). This is the leverage effect. The return will be –10%.

8. **a.** \( F_0 = S_0 (1 + r_f) = 150 \times 1.06 = 159 \)

**b.** \( F_0 = S_0 (1 + r_f)^3 = 150 \times 1.06^3 = 178.65 \)

**c.** \( F_0 = 150 \times 1.08^3 = 188.96 \)

9. **a.** Take a short position in T-bond futures, to offset interest rate risk. If rates increase, the loss on the bond will be offset to some extent by gains on the futures.

**b.** Again, a short position in T-bond futures will offset the interest rate risk.

**c.** You want to protect your cash outlay when the bond is purchased. If bond prices increase, you will need extra cash to purchase the bond. Thus, you should take a long futures position that will generate a profit if prices increase.

10. \( F_0 = S_0 \times (1 + r_f - d) = 1,500 \times (1 + 0.05 - 0.02) = 1,545 \)

11. The put-call parity relation states that:

\[
P = C - S_0 + \frac{X}{(1 + r_f)^T}
\]

If \( X = F \), then: \( P = C - S_0 + \frac{F}{(1 + r_f)^T} \)

But spot-futures parity tells us that:

\[
F = S_0 (1 + r_f)^T
\]

Substituting, we find that:

\[
P = C - S_0 + \frac{[S_0 (1 + r_f)^T]/(1 + r_f)^T}{C - S_0 + S_0} \text{ which implies that } P = C.
\]
12. According to the parity relation, the proper price for December futures is:
\[ F_{Dec} = F_{June}(1 + r_f)^{1/2} = 846.30 \times 1.05^{1/2} = 867.20 \]
The actual futures price for December is low relative to the June price. You should take
a long position in the December contract and short the June contract.

13. a. \[ 120 \times 1.06 = 127.20 \]
b. The stock price falls to: \[ 120 \times (1 - 0.03) = 116.40 \]
The futures price falls to: \[ 116.4 \times 1.06 = 123.384 \]
The investor loses: \( (127.20 - 123.384) \times 1,000 = 3,816 \)
c. The percentage loss is: \( \frac{3,816}{12,000} = 0.318 = 31.8\% \)

14. a. The initial futures price is \[ F_0 = 1300 \times (1 + 0.005 – 0.002)^{12} = 1,347.58 \]
In one month, the futures price will be:
\[ F_0 = 1320 \times (1 + 0.005 – 0.002)^{11} = 1,364.22 \]
The increase in the futures price is 16.64, so the cash flow will be:
\[ 16.64 \times 250 = 4,160.00 \]
b. The holding period return is: \( \frac{4,160.00}{13,000} = 0.3200 = 32.00\% \)

15. The treasurer would like to buy the bonds today, but cannot. As a proxy for this
purchase, T-bond futures contracts can be purchased. If rates do in fact fall, the treasurer
will have to buy back the bonds for the sinking fund at prices higher than the prices at
which they could be purchased today. However, the gains on the futures contracts will
offset this higher cost to some extent.

16. The parity value of \( F \) is: \[ 1,300 \times (1 + 0.04 – 0.01) = 1,339 \]
The actual futures price is 1,330, too low by 9.

\[
\begin{array}{ccc}
\text{Arbitrage Portfolio} & \text{CF now} & \text{CF in 1 year} \\
\text{Short Index} & 1,300 & -S_T - (0.01 \times 1,300) \\
\text{Buy Futures} & 0 & S_T - 1,330 \\
\text{Lend} & -1,300 & 1,300 \times 1.04 \\
\text{Total} & 0 & 9 \\
\end{array}
\]
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17. a. Futures prices are determined from the spreadsheet as follows:

<table>
<thead>
<tr>
<th>Spot Futures Parity and Time Spreads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot price</td>
</tr>
<tr>
<td>Income yield (%)</td>
</tr>
<tr>
<td>Interest rate (%)</td>
</tr>
<tr>
<td>Today's date</td>
</tr>
<tr>
<td>Maturity date 1</td>
</tr>
<tr>
<td>Maturity date 2</td>
</tr>
<tr>
<td>Maturity date 3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spot price</th>
<th>Futures prices versus maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,500.00</td>
<td></td>
</tr>
<tr>
<td>1,502.67</td>
<td></td>
</tr>
<tr>
<td>1,508.71</td>
<td></td>
</tr>
<tr>
<td>1,519.79</td>
<td></td>
</tr>
</tbody>
</table>

Time to maturity 1 | 0.12 |
Time to maturity 2 | 0.39 |
Time to maturity 3 | 0.88 |

**LEGEND:**

Enter data
Value calculated
See comment

b. The spreadsheet demonstrates that the futures prices now decrease with increased time to maturity:

<table>
<thead>
<tr>
<th>Spot Futures Parity and Time Spreads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot price</td>
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<td>Today's date</td>
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<tr>
<td>Maturity date 1</td>
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<td>Maturity date 2</td>
</tr>
<tr>
<td>Maturity date 3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spot price</th>
<th>Futures prices versus maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,500.00</td>
<td></td>
</tr>
<tr>
<td>1,498.20</td>
<td></td>
</tr>
<tr>
<td>1,494.15</td>
<td></td>
</tr>
<tr>
<td>1,486.78</td>
<td></td>
</tr>
</tbody>
</table>

Time to maturity 1 | 0.12 |
Time to maturity 2 | 0.39 |
Time to maturity 3 | 0.88 |

**LEGEND:**

Enter data
Value calculated
See comment

18. a. The current yield for Treasury bonds (coupon divided by price) plays the role of the dividend yield.

b. When the yield curve is upward sloping, the current yield exceeds the short rate. Hence, T-bond futures prices on more distant contracts are lower than those on near-term contracts.
19. a. The cash flows and action plan are as follows:

<table>
<thead>
<tr>
<th>Action</th>
<th>Now</th>
<th>T₁</th>
<th>T₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long futures with maturity T₁</td>
<td>0</td>
<td>P₁ – F(T₁)</td>
<td>0</td>
</tr>
<tr>
<td>Short futures with maturity T₂</td>
<td>0</td>
<td>0</td>
<td>F(T₂) – P₂</td>
</tr>
<tr>
<td>Buy asset at T₁, sell at T₂</td>
<td>0</td>
<td>−P₁</td>
<td>+P₂</td>
</tr>
<tr>
<td>At T₁, borrow F(T₁)</td>
<td>0</td>
<td>F(T₁)</td>
<td>−F(T₁) × (1 + rₓ) (T₂−T₁)</td>
</tr>
<tr>
<td>Total</td>
<td>0</td>
<td>0</td>
<td>F(T₂) – F(T₁) × (1 + rₓ) (T₂−T₁)</td>
</tr>
</tbody>
</table>

b. Since the T₂ cash flow is riskless and the net investment was zero, then any profits represent an arbitrage opportunity.

c. The zero-profit no-arbitrage restriction implies that

\[ F( T₂ ) = F( T₁ ) × (1 + rₓ) (T₂−T₁) \]

**CFA PROBLEMS**

1. a. The strategy that would take advantage of the arbitrage opportunity is a “reverse cash and carry.” A reverse cash and carry opportunity results when the following relationship does not hold true:

\[ F₀ ≥ S₀ (1 + C) \]

If the futures price is less than the spot price plus the cost of carrying the goods to the futures delivery date, then an arbitrage opportunity exists. A trader would be able to sell the asset short, use the proceeds to lend at the prevailing interest rate, and then buy the asset for future delivery. At the future delivery, the trader would then collect the proceeds of the loan with interest, accept delivery of the asset, and cover the short position in the commodity.

b. The cash flows are as follows:

<table>
<thead>
<tr>
<th>Action</th>
<th>Now</th>
<th>One year from now</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell the spot commodity short</td>
<td>+$120.00</td>
<td>−$125.00</td>
</tr>
<tr>
<td>Buy the commodity futures expiring in 1 year</td>
<td>$0.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>Contract to lend $120 at 8% for 1 year</td>
<td>−$120.00</td>
<td>+$129.60</td>
</tr>
<tr>
<td>Total cash flow</td>
<td>$0.00</td>
<td>+$4.60</td>
</tr>
</tbody>
</table>
2. a. The call option is distinguished by its asymmetric payoff. If the Swiss franc rises in value, then the company can buy francs for a given number of dollars to service its debt, and thereby put a cap on the dollar cost of its financing. If the franc falls, the company will benefit from the change in the exchange rate. The futures and forward contracts have symmetric payoffs. The dollar cost of the financing is locked in regardless of whether the franc appreciates or depreciates. The major difference from the firm’s perspective between futures and forwards is in the mark-to-market feature of futures. The consequence of this is that the firm must be ready for the cash management issues surrounding cash inflows or outflows as the currency values and futures prices fluctuate.

b. The call option gives the company the ability to benefit from depreciation in the franc, but at a cost equal to the option premium. Unless the firm has some special expertise in currency speculation, it seems that the futures or forward strategy, which locks in a dollar cost of financing without an option premium, may be the better strategy.

3. The important distinction between a futures contract and an options contract is that the futures contract is an obligation. When an investor purchases or sells a futures contract, the investor has an obligation to either accept or deliver, respectively, the underlying commodity on the expiration date. In contrast, the buyer of an option contract is not obligated to accept or deliver the underlying commodity but instead has the right, or choice, to accept delivery (for call holders) or make delivery (for put holders) of the underlying commodity anytime during the life of the contract.

Futures and options modify a portfolio’s risk in different ways. Buying or selling a futures contract affects a portfolio’s upside risk and downside risk by a similar magnitude. This is commonly referred to as symmetrical impact. On the other hand, the addition of a call or put option to a portfolio does not affect a portfolio’s upside risk and downside risk to a similar magnitude. Unlike futures contracts, the impact of options on the risk profile of a portfolio is asymmetric.

4. a. The investor should sell the forward contract to protect the value of the bond against rising interest rates during the holding period. Because the investor intends to take a long position in the underlying asset, the hedge requires a short position in the derivative instrument.

b. The value of the forward contract on expiration date is equal to the spot price of the underlying asset on expiration date minus the forward price of the contract:

\[ \text{$978.40 - $1,024.70 = -$46.30} \]

The contract has a negative value. This is the value to the holder of a long position in the forward contract. In this example, the investor should be short the forward contract, so that the value to this investor would be $+$46.30 since this is the cash flow the investor expects to receive.
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c. The value of the combined portfolio at the end of the six-month holding period is:
   \[ \$978.40 + \$46.30 = \$1,024.70 \]
The change in the value of the combined portfolio during this six-month period is:
   \[ \$24.70 \]
The value of the combined portfolio is the sum of the market value of the bond
and the value of the short position in the forward contract. At the start of the six-
month holding period, the bond is worth \$1,000 and the forward contract has a
value of zero (because this is not an off-market forward contract, no money
changes hands at initiation). Six months later, the bond value is \$978.40 and the
value of the short position in the forward contract is \$46.30, as calculated in part
(b).
The fact that the combined value of the long position in the bond and the short
position in the forward contract at the forward contract’s maturity date is equal to
the forward price on the forward contract at its initiation date is not a coincidence.
By taking a long position in the underlying asset and a short position in the
forward contract, the investor has created a fully hedged (and hence risk-free)
position, and should earn the risk-free rate of return. The six-month risk-free rate
of return is 5.00% (annualized), which produces a return of \$24.70 over a six-
month period:
   \[ \left( \$1,000 \times 1.05^{(1/2)} \right) - \$1,000 = \$24.70 \]
These results support VanHusen’s statement that selling a forward contract on the
underlying bond protects the portfolio during a period of rising interest rates. The
loss in the value of the underlying bond during the six month holding period is offset
by the cash payment made at expiration date to the holder of the short position in the
forward contract; that is, a short position in the forward contract protects (hedges) the
long position in the underlying asset.

5. a. Accurate. Futures contracts are marked to the market daily. Holding a short position
on a bond futures contract during a period of rising interest rates (declining bond
prices) generates positive cash inflow from the daily mark to market. If an investor in
a futures contract has a long position when the price of the underlying asset increases,
then the daily mark to market generates a positive cash inflow that can be reinvested.
Forward contracts settle only at expiration date and do not generate any cash flow
prior to expiration.

b. Inaccurate. According to the cost of carry model, the futures contract price is adjusted
upward by the cost of carry for the underlying asset. Bonds (and other financial
instruments), however, do not have any significant storage costs. Moreover, the cost
of carry is reduced by any coupon payments paid to the bondholder during the life of
the futures contract. Any “convenience yield” from holding the underlying bond also
reduces the cost of carry. As a result, the cost of carry for a bond is likely to be
negative.