Futures and Forward Contracts

1. Follow the price of an S&P 500 Futures contract (traded on the CME) for 5 days. Use the contract that calls for delivery in a month or so. Assume that the initial margin is $20,000 and the variation margin is $15,000 per contract. Make a table to track the market value of the contract, the daily profit (loss), your margin account balance, and your ‘cash in’ (when there are margin calls). Do this twice: once for a long position in 5 contracts, and the second time for a short position in two contracts. What are the profits from the two strategies?

2. Suppose the current spot price of gold is $387 per ounce. There is a one-year forward contract for 1,000 ounces of gold. The one-year forward price is $460 per ounce. The interest rate is 12% per year (for borrowing and lending).
   a) Are these equilibrium prices? If not, show the arbitrage profit from trading one forward contract. If so, explain why.
   b) Solve for the equilibrium forward price, taking the spot price as given.
   c) Solve for the equilibrium spot price, taking the forward price as given.
   d) Solve for the equilibrium cost of carry (interest rate), taking the forward and spot prices of gold as given.

3. Suppose the current spot price of gold is $205 per ounce. There is a one-year forward contract for 1,000 ounces of gold. The one-year forward price is $224 per ounce. To borrow, it will cost you 10.1% per year. To lend, you will earn an interest rate of 8.7% per year.
   a) Are these equilibrium prices? If not, show the arbitrage profit from trading one forward contract. If so, explain why.
   b) Solve for the equilibrium futures price (bounds), taking the spot price as given.
   c) Solve for the equilibrium spot price (bounds), taking the forward price as given.
   d) What happens to the equilibrium bounds for the forward price if the spread between the lending and borrowing rate increases? For example, suppose the lending rate is only 8.5%. Are the equilibrium forward price bounds wider than in b?

4. Suppose that, when the stock market closes Friday, Feb 18, the S&P 500 is valued at 1355.00. At the same time, the Mar00 futures contract on the S&P 500 is trading at 1361.08. Assume that interest is compounded daily and that we have perfect markets (same borrowing and lending rates, etc.). Treat the futures contract as a forward contract (i.e., ignore the implications of a margin account).
   a) Just before the stock market opens in the morning on Tuesday, Feb 22, you see that the S&P 500 futures are trading at 1358.22. If the futures remain at this level, what change do you expect to see in the stock market (specifically in the S&P 500 index)?
   b) Just before the stock market opens in the morning on Tuesday, Feb 22, you see that the S&P 500 futures are trading at 1378.30. If the futures remain at this level, what change do you expect to see in the stock market (specifically in the S&P 500 index)?
   c) What would ‘fair value’ be for the futures contract? In other words, if the stock market were to open at exactly the same price on Tuesday morning as it closed on Friday afternoon, what would the futures contract value be?

5. Suppose the current spot price of gold has a bid price of $401 per ounce and an ask price of $403. There is a one year forward contract for 1,000 ounces of gold. The one-year forward price is bid at $431 per ounce and offered (asked) at $433 per ounce. To borrow, it will cost you 8.4% per year. To lend, you will earn an interest rate of 8.3% per year.
   a) Are these equilibrium prices? If not, show the arbitrage profit from trading one forward contract. If so, explain why.
   b) Solve for the equilibrium futures price (bounds), taking the spot price as given.
   c) Solve for the equilibrium spot price (bounds), taking the forward price as given.
   d) Show that a decrease in the bid-ask spread in the spot market will decrease the bounds on the equilibrium forward price. To do this, assume that the bid price increases by $0.50 and the ask price decreases by $0.50.
Foreign Exchange

6. Suppose the current spot exchange rate between a US$ and UK£ is 1.6746 US$/UK£. The spot exchange rate between US$ and German Mark (DM) is 0.5785 US$/DM. In equilibrium, what should the direct exchange rate be for UK£ to DM?

7. Look up (in the Wall Street Journal or, possibly, the AJC) the exchange rate between the US$ and the following countries’ currency: Canada, France, Japan, and Switzerland. Fill in the following table for direct conversion of currencies (you should check that your obtained answers are very close to the cross rates reported in the Wall Street Journal under ‘Key Currency Cross Rates’).

<table>
<thead>
<tr>
<th>Country</th>
<th>Canada</th>
<th>France</th>
<th>Japan</th>
<th>Switzerland</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>1.0000</td>
<td>$Ca/$Ca</td>
<td>$Ca/FF</td>
<td>$Ca/¥</td>
</tr>
<tr>
<td>France</td>
<td>FF/$Ca</td>
<td>1.0000</td>
<td>FF/FF</td>
<td>FF/¥</td>
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<tr>
<td>Japan</td>
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<td>¥/¥</td>
</tr>
<tr>
<td>Switzerland</td>
<td>SF/$Ca</td>
<td>SF/FF</td>
<td>SF/¥</td>
<td>1.0000</td>
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</tbody>
</table>

8. Suppose that you have the following information:
   i. Perfect markets. I.e., no transactions costs and no restrictions on short-selling.
   ii. The spot exchange rate is 134.25 ¥/US$.
   iii. The 1-year forward exchange rate is 129.50 ¥/US$.
   iv. The 1-year return on $-denominated riskfree bonds in the U.S. is 4.85%.
   v. The 1-year return on ¥-denominated riskfree bonds in Japan is 4.30%.

   Answer the following questions:
   a) Is there an arbitrage opportunity? If so, show how you would take advantage of it.
   b) Assuming that i.-iv. are correct, what is the equilibrium (no arbitrage) rate of return on ¥-denominated riskfree Japanese bonds?
   c) Assuming that i.-iii. and v. are correct, what is the equilibrium (no arbitrage) rate of return on $-denominated riskfree U.S. bonds?
   d) Assuming that i., ii., iv. and v. are correct, what is the equilibrium (no arbitrage) forward price of US$ in terms of ¥? ¥ in terms of US$?
   e) Assuming that i. and iii.-v. are correct, what is the equilibrium (no arbitrage) spot price of the US$ in terms of the ¥? ¥ in terms of US$?

9. Suppose that you have the following information:
   i. Perfect markets. I.e., no transactions costs and no restrictions on short-selling.
   ii. The spot exchange rates are 135.50 ¥/US$ and 1.6750 US$/UK£.
   iii. The 1-year forward exchange rates are 134.90 ¥/US$ and 1.6600 US$/UK£.
   iv. The 1-year return on £-denominated riskfree bonds in the U.K. is 5.95%.
   v. The 1-year return on ¥-denominated riskfree bonds in Japan is 4.10%.

   Is there an arbitrage opportunity? If so, show how you would take advantage of it by showing the exact arbitrage transactions that you would use.
   Hint: You have no information on the direct conversion rates between £ and ¥. Therefore, you may not assume any exchange rates other than those given.