Option Payoffs

Problems 1 through 10: Assume that the stock is currently trading at $20 per share and options and bonds have the prices given in the table below. Depending on the strike price (X) of the option or the face value (FV) of the bond, do the following:

a) Sketch the payoff and profit diagrams for the strategy.
b) Describe the profitable range in terms of the price of the underlying security.
c) Describe the maximum potential profits and losses.

<table>
<thead>
<tr>
<th>X or FV</th>
<th>Call Premium</th>
<th>Put Premium</th>
<th>Bond Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ 5</td>
<td>$ 15.60</td>
<td>$ 0.10</td>
<td>$ 4.50</td>
</tr>
<tr>
<td>10</td>
<td>11.80</td>
<td>0.90</td>
<td>9.00</td>
</tr>
<tr>
<td>15</td>
<td>9.00</td>
<td>2.60</td>
<td>13.60</td>
</tr>
<tr>
<td>20</td>
<td>6.90</td>
<td>5.00</td>
<td>18.10</td>
</tr>
<tr>
<td>25</td>
<td>5.40</td>
<td>8.00</td>
<td>22.60</td>
</tr>
<tr>
<td>30</td>
<td>4.30</td>
<td>11.40</td>
<td>27.10</td>
</tr>
<tr>
<td>35</td>
<td>3.40</td>
<td>15.10</td>
<td>31.70</td>
</tr>
</tbody>
</table>

1. Short one call, X=10; Long one bond, FV=10.
2. Long one call, X=20; Short one call, X=30.
3. Long one share of stock; short one call, X=20.
4. Long one put, X=15; Long one call, X=25.
5. Short one put, X=15; Long one call, X=20.
6. Long one put, X=10; Short one put, X=20; Long one call, X=30.
7. Short one put, X=10; Long one put, X=20; Short one call, X=20.
8. Long two calls, X=25; Short one call, X=10; Long one bond, FV=10.
9. Long one put, X=10; Short one put, X=15; Short one call, X=25; Long one call, X=30.
10. Short one share of stock; Short one put, X=20; Long one bond, FV=25; Long one call, X=25.

Problems 11 through 16: Describe (as I have in 1-10) the strategy depicted by each payoff diagram.
Arbitrage

17. Suppose that there are two call options written on the same underlying asset. Call A has a strike price of $50 and Call B has a strike price of $60. Call A’s premium is $5.25 and Call B’s premium is $6.50. Describe the arbitrage opportunity that takes advantage of these prices and prove that this is an arbitrage strategy.

*Hint:* draw the payoff diagrams.

18. Suppose that there are two put options written on the same underlying asset. Put Y has a strike price of $30 and Put Z has a strike price of $40. Put Y’s premium is $3.25 and Put Z’s premium is $2.50. Describe the arbitrage opportunity that takes advantage of these prices and prove that this is an arbitrage strategy.

*Hint:* draw the payoff diagrams.

19. Suppose that you know that strike price X2 is greater than strike price X1 (i.e., X1 < X2). Generalizing the results from problems 17 and 18, what conditions must hold in equilibrium for the call option premiums in terms of X1 and X2 (assuming the same underlying asset and time to expiration)? That is, which call option should have the higher premium? What about the put option premiums?

20. Suppose that a stock’s current price is $88 and the riskfree interest rate is 10%. One-year European call options with a strike price of $90 trade for $3.50. Is there an arbitrage opportunity here? If so, prove it.

21. Suppose that a stock’s current price is $78. Options written on the stock have a strike price of $80 and expire in 6 months. The price of the call option is $6 and the price of the put option is $8. The riskfree interest rate is 5% per year.
   a) Show how you could take advantage of these prices to earn an arbitrage profit.
   b) Taking the stock price, call price, and interest rate as given, what is the equilibrium put premium?
   c) Taking the stock price, put price, and interest rate as given, what is the equilibrium call premium?
   d) Taking the stock price, put price, and interest rate as given, what is the equilibrium stock price?
   e) Taking the stock price, put price, and call price as given, what is the equilibrium riskfree rate?

22. Look up a stock in the *Wall Street Journal* that has options actively traded on it (you should start by looking in the “options” section of the WSJ). If possible, find a stock that will not pay a dividend prior to the expiration of options and use options with at least 1 month to expiration.
   a) Using put-call parity, find the value of a synthetic put.
   b) Using put-call parity, find the value of a synthetic call.
   c) How far off are your premiums for the synthetic options versus the actual options? Is there an arbitrage opportunity? Why or why not?

23. Suppose that there are options written on ABC stock with two strike prices: X1=$60 and X2=$65. The current premiums of the call options are C(X1)=$10 and C(X2)=$6 (i.e., the premium on the call option with a strike price of X1=$60 is $10). The current put premiums are P(X1)=$3.50 and P(X2)=$4.50. The riskfree interest rate is 6%. All options expire in one year. Show how you could take advantage of these prices to earn an arbitrage profit.

*Hint:* Note that you cannot trade the ABC stock directly, since you don’t know what its price is. There will be five possible outcomes at expiration.

24. Suppose that there are options written on XYZ stock with two strike prices: X1=$100 and X2=$110. One share of XYZ stock currently trades for $105. The current premiums of the call options are C(X1) = $25 and C(X2) = $15 (i.e., the premium on the call option with a strike price of X1=$100 is $25). The current put premiums are P(X1) = $12 and P(X2) = $10. Show how you could take advantage of these prices to earn an arbitrage profit.

*Hint:* This problem is similar in nature to the previous problem. What “price” don't you know?
Binomial Option Pricing Model

Problems 25 through 31. The definitions of the variables are: \( S \) = current stock price; \( X \) = strike price of the options; \( U \) = multiplier for up movements in the stock price each period; \( r \) = interest rate per period; \( T \) = number of periods until expiration.

<table>
<thead>
<tr>
<th>Problem</th>
<th>( S )</th>
<th>( X )</th>
<th>( U )</th>
<th>( r )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>53</td>
<td>50</td>
<td>1.08</td>
<td>4%</td>
<td>1</td>
</tr>
<tr>
<td>26</td>
<td>51</td>
<td>50</td>
<td>1.08</td>
<td>4%</td>
<td>1</td>
</tr>
<tr>
<td>27</td>
<td>53</td>
<td>55</td>
<td>1.08</td>
<td>4%</td>
<td>1</td>
</tr>
<tr>
<td>28</td>
<td>53</td>
<td>50</td>
<td>1.12</td>
<td>4%</td>
<td>1</td>
</tr>
<tr>
<td>29</td>
<td>53</td>
<td>50</td>
<td>1.08</td>
<td>6%</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td>53</td>
<td>50</td>
<td>1.08</td>
<td>4%</td>
<td>2</td>
</tr>
<tr>
<td>31</td>
<td>53</td>
<td>55</td>
<td>1.08</td>
<td>4%</td>
<td>2</td>
</tr>
</tbody>
</table>

25. Use the single-period binomial option pricing model to answer the following questions.
   a) What is the equilibrium call premium?
   b) What is the equilibrium put premium?

26. Use the single-period binomial option pricing model to answer the following questions.
   a) What is the equilibrium call premium?
   b) What is the equilibrium put premium?
   c) Compare your answers with #25. What can you conclude about how option prices change with a change in the price of the underlying stock?

27. Use the single-period binomial option pricing model to answer the following questions.
   a) What is the equilibrium call premium?
   b) What is the equilibrium put premium?
   c) Compare your answers with #25. What can you conclude about how option prices change with a change in the strike price of the option?

28. Use the single-period binomial option pricing model to answer the following questions.
   a) What is the equilibrium call premium?
   b) What is the equilibrium put premium?
   c) Compare your answers with #25. What can you conclude about how option prices change with a change in the volatility of the underlying stock?

29. Use the single-period binomial option pricing model to answer the following questions.
   a) What is the equilibrium call premium?
   b) What is the equilibrium put premium?
   c) Compare your answers with #25. What can you conclude about how option prices change with a change in the riskfree rate?

30. Use the two-period binomial option pricing model to answer the following questions.
   a) What is the equilibrium European call premium?
   b) What is the equilibrium European put premium?
   c) What is the equilibrium American call premium?
   d) What is the equilibrium American put premium?
   e) Compare your answers with #25. What can you conclude about how option prices change with a change in the time to expiration?
31. Use the two-period binomial option pricing model to answer the following questions.
   a) What is the equilibrium European call premium?
   b) What is the equilibrium European put premium?
   c) What is the equilibrium American call premium?
   d) What is the equilibrium American put premium?
   e) From your answers above, what can you conclude about the value of the option to exercise early for an American call? For an American put?

32. Suppose the market price for the call option in #25 is $4.75. Show an arbitrage strategy that takes advantage of this mispricing.
   **Hint:** This is not a put-call parity question. Do not assume that a put option exists. The only assets in the market are a stock, a bond (riskless borrowing and lending), and a call option.

33. Suppose the market price for the call option in #25 is $5.375. Show an arbitrage strategy that takes advantage of this mispricing.
   **Hint:** This is not a put-call parity question. Do not assume that a put option exists. The only assets in the market are a stock, a bond (riskless borrowing and lending), and a call option.

34. Suppose the market price for the put option in #25 is $0.50. Show an arbitrage strategy that takes advantage of this mispricing.
   **Hint:** This is not a put-call parity question. Do not assume that a call option exists. The only assets in the market are a stock, a bond (riskless borrowing and lending), and a put option.

35. Suppose the market price for the call option in #31 is $4.00. Show an arbitrage strategy that takes advantage of this mispricing.
   **Hint:** This is not a put-call parity question. Do not assume that a call option exists. The only assets in the market are a stock, a bond (riskless borrowing and lending), and a call option. Also, you must illustrate your cash flows at every date and in every possible state of the world at each date. At time 1, there are two possible states. At time 2, there are three possible states. Be sure to describe how your portfolio is rebalanced in each state at time 1.

**Black-Scholes Option Pricing Model**

Problems 36 through 42: View these problems in terms of the Black-Scholes option pricing model. You may want to use the Black-Scholes spreadsheet that I have provided on the course web site. You will need to program the Black-Scholes model into your own spreadsheet in order to answer questions 38 and 39.

36. Suppose the value of the underlying asset increases, what impact does this have on...
   a) the call premium? Why?
   b) the put premium? Why?

37. Suppose the strike price of the option increases, what impact does this have on...
   a) the call premium? Why?
   b) the put premium? Why?

38. Suppose the volatility of the underlying asset increases, what impact does this have on...
   a) the call premium? Why?
   b) the put premium? Why?

39. Suppose the riskfree interest rate decreases, what impact does this have on...
   a) the call premium? Why?
   b) the put premium? Why?
40. Suppose the time to expiration of an option decreases (as would happen as the option would get closer to expiration - while no other key variables change), what impact does this have on...
   a) the call premium? Why?
   b) the put premium? Why?

41. Suppose that a stock currently trades for $76.125. You know that the true standard deviation of the stock's returns is exactly 41.5%. The call option expires in 70 days and has a strike price of $75. The riskfree rate of return is 4.1%. The market price of the call is currently $5.50.
   a) Is the market's assessment of the volatility of the underlying asset too high or too low? Why?
   b) What is the market's estimate of the volatility of the stock, according to the market prices of the call and the stock?
   *Hint:* You might need to use the 'Solver' function in Microsoft Excel in order to 'back out' the implied volatility.

42. Use the *Wall Street Journal* or an internet quote service to obtain data for the following questions. For the riskfree rate of return, use the ask yield on the T-bill that matures in the same month (or closest) as the month of expiration for the options.
   a) Find an options contract on a 'Blue-chip' company (such as Boeing or General Electric). Using the market prices, determine the implied volatility (the sigma) that the market is using for the call option that is closest to being at-the-money and that expires in one to three months. Try not to use a firm that pays dividends.
   *Hint:* You will need to use the 'Solver' function in Microsoft Excel in order to 'back out' the implied volatility.
   b) Find an options contract on an internet company. Using the market prices, determine the implied volatility (the sigma) that the market is using for the call option that is closest to being at-the-money and that expires in one to three months.
   *Hint:* You will need to use the 'Solver' function in Microsoft Excel in order to 'back out' the implied volatility.
   c) Compare your answers to a and b above. Is this what you would expect? Why or why not?