Asymmetric Information and Stock Return Cross-Autocorrelations

Dan Bernhardt *
University of Illinois at Urbana-Champaign

Reza S. Mahani †
Georgia State University

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Abstract

Using an asset pricing model under asymmetric information, we show that asymmetric lead-lag patterns in stock returns cannot be solely explained by information asymmetry. Additional frictions are necessary to produce asymmetry in return cross-autocorrelations.

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Cross-autocorrelation patterns in stock returns have been studied extensively in the past two decades. It is now well-known that in the short-run, cross-autocorrelations are positive and more significant than own-autocorrelations. Moreover, return cross-autocorrelations across portfolios formed on financial variables such as firm size (Lo and MacKinlay (1988)), volume (Chordia and Swaminathan (2000)), analyst coverage (Brennan, Jegadeesh and Swaminathan (1993)), and institutional ownership (Badrinath, Kale and Noe (1995), Sias and Starks (1997)), show significant asymmetry. For example, current returns on portfolios of large stocks better predict future returns on portfolios of small stocks than the converse (e.g., Lo and MacKinlay 1990). Researchers such as Chan (1993) have posited that this asymmetric pattern in cross-autocorrelations between large stocks and small may arise because there is better public information about large stocks, possibly due to lower marginal costs of producing information (e.g., Ho and Michaely 1988). Indeed, Chan (1993) derives precisely this result by endowing market makers with noisy signals of “their” stock’s value, and assumes that they set prices equal to the expected value of their asset given their noisy signal. Heterogeneity in signals’ noise immediately produces the asymmetric pattern in cross-autocorrelations: The price changes for stocks with less noisy signals are more strongly correlated with future price changes of stocks with noisier signals than vice versa, as for the latter stocks, information is incorporated into prices more slowly.

This paper shows that endogenizing market maker information by endogenizing trader strategies changes this conclusion. To do this, we extend the classical competitive dealership model of Kyle (1985), so that innovations to stock values that are not public information reflect both a common market component and an idiosyncratic component. For example, stocks that are less heavily followed by analysts, typically small stocks, may have relatively larger idiosyncratic components. Informed agents receive signals that are a general linear combination of these two components, and private information is short lived as in Admati and Pfleiderer (1988).

In this setting, when market makers for a stock only observe own stock order flow, then due to the common market component to innovations, information in the returns of one stock will predict returns of other stocks, even though a stock return’s own autocorrelation is zero due to the martingale property of prices. One might think that the cross-autocorrelations will be asymmetric, as agents with information about the common market shock trade stocks with relatively greater exposure to the market innovation more aggressively, while speculation on idiosyncratic components is greater in stocks where that component is relatively larger.

We prove that this is not so—lagged returns of high market component stocks do not better predict future returns of low market component stocks, than the converse. Even though lagged returns of high market component stocks contain more information, they are predicting a noisier variable (returns of a low market component stock). In equilibrium, speculators choose their trading intensities so that the two effects exactly cancel out. In turn, it follows that the cross-asset predictive powers of different stocks—as captured by the $R^2$ of a regression of current returns on past returns—are the same. These results are
robust—they hold for varying numbers of informed traders and correlation in liquidity across stocks, and can be extended to allow for noisy signals.

1 The model

We consider a discrete-time framework with a finite number of assets indexed by \( s = 1, \ldots, S \). Asset values evolve according to

\[
V_t^s = V_{t-1}^s + \sigma_s g_t^s + \omega_s m_t \quad g_t^s, m_t \sim \mathcal{N}(0, 1), \quad s = 1, \ldots, S
\]

where the innovations \( g_t^s \) and \( m_t \) are jointly normally and independently distributed of each other and all past innovations. Here, \( g^s \) is the component of the innovation to the value of asset \( s \) that is specific to the asset, and \( m_t \) is a market component common to all stocks. In this context, a ‘large’ stock might be one with a relatively greater weight, \( \omega_s \), on the market component, \( m_t \), while a ‘small’ stock may have relatively greater exposure to stock-specific private information, \( \sigma_s g_t^s \). This might be the case if more of the idiosyncratic information about a large stock is already public information, for example, due to greater analyst coverage.

Risk-neutral informed traders have private information about the end-of-period asset values. Asset values \( V_t^s \) are revealed publicly at the end of each period. \( n_s \) informed agents receive the signal \( \epsilon_t^s \) related to the end-of-period value of asset \( s \):

\[
\epsilon_t^s = \psi^s g_t^s + \phi^s m_t.
\]

For example, a signal for a large stock may contain less information about the stock-specific innovation, \( g_t^s \), and more information about \( m_t \). The informational setting is quite general. For instance, by introducing a non-traded or fictitious asset \( S \) whose signal has a weight of zero on the idiosyncratic component, i.e., \( \psi^S = 0 \), it allows for a ‘pure’ signal about the market component, \( m_t \). An agent who sees signal \( s \) can trade any stock in the set \( S \) of traded assets. Let \( x_t^s \) be the trade of stock \( s \) by an individual who sees signal \( s \).

In addition to informed traders, there are liquidity/noise traders whose order in stock \( s \) at date \( t \), is independently and normally distributed, \( u_t^s \sim \mathcal{N}(0, \eta_s) \). The agents trade in a competitive dealership market made by risk neutral market makers. A market maker for stock \( s \) observes only the net order flow for asset \( s \). Competition induces market makers to set the period price of asset \( s \) equal to its expected value given the net order flow for asset \( s \).

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1 We can write a multiplicative version of the model that obtains the results for returns at the cost of more algebra. Instead, we write our model in price changes and our characterizations are in terms of price changes rather than returns.

2 Dennis and Strickland (2005) document empirically that size is negatively correlated with idiosyncratic volatility.

3 Our results extend to noisy signals.

4 All results extend to allow for cross-asset correlation in liquidity trade.
The timing is as follows. At the beginning of period $t$, speculators observing signal $\epsilon_t^s$, $\forall s' \in S$, submit orders $x_t^{s',s}$, and liquidity traders submit their orders $u_t^s$ for asset $s \in S$. Next, given net order flow, $Z_t^s \equiv \sum_{s'=1}^{S} n_{s'} x_t^{s',s} + u_t^s$, each market maker for asset $s$ sets the price $P_t^s$ at which they fill their orders. Finally, trading profits and losses are realized.

The next proposition shows that there is a unique equilibrium in which prices are linear functions of the net order flows and trading strategies are linear functions of signals.

**Proposition 1** In the unique linear equilibrium, the price for stock $s \in S$ takes the form $P_t^s = V_{t-1}^s + \lambda_s Z_t^s$, and the trade of an agent with signal $\epsilon_t^{s'}$ for stock $s$ solves $\lambda_s x_t^{s',s'} = k_{s',s} \epsilon_t^{s'}$. All proofs, as well as the explicit solutions for $k_{s',s}$ and $\lambda_s$, are given in the appendix.

**Cross-autocorrelations.** We next present our equivalence results. We look at price changes $P_{t+1}^s - P_t^s \equiv \Delta_{t+1} P^s$ and their cross-autocorrelations:

**Proposition 2** Regardless of the asset characteristics, cross-auto covariances between any two assets $s$ and $s'$ are symmetric:

$$E[\Delta_{t+1} P^s \Delta_t P^{s'}] = E[\Delta_{t+1} P^{s'} \Delta_t P^s].$$

To understand our equivalence result for cross-autocorrelations, consider two assets, $l$ and $L$, where asset $l$ has relatively more unrevealed information about idiosyncratic innovations, $\sigma_l > \sigma_L$, and the common component enters more prominently into the value of asset $L$, i.e., $\omega_l < \omega_L$. Despite these differences, the two cross-autocorrelations are the same. Most transparently, when signals are pure so that speculators either see the market shock or the idiosyncratic shock for a stock we have $E[\Delta_{t+1} P^s \Delta_t P^{s'}] = E[\Delta_{t+1} P^{s'} \Delta_t P^s] = n_{m} \omega^{s'} \omega^s$, where $n_{m}$ speculators observe the pure market shock. In particular, neither the number of speculators $n^s$ seeing the idiosyncratic shock $g^s$, nor the magnitude of the idiosyncratic shock affect these cross-autocorrelations.

A corollary of Proposition 2 is that for cross-asset returns, the predictive powers of these two stocks—as captured by the $R^2$ of a regression of $\Delta_{t+1} P^l$ on $\Delta_t P^L$ (and $\Delta_{t+1} P^L$ on $\Delta_t P^l$)—are the same. The $R^2$ of such a regression is $\frac{(E[\Delta_{t+1} P^l \Delta_t P^L])^2}{E[\Delta_{t+1} P^l]^2 E[\Delta_t P^L]^2}$. What underlies this result is that while lagged price changes of stock $L$ with more exposure to the market component contain more common information than do lagged changes of the prices of stock $l$, and hence are better predictors, there is more idiosyncratic uncertainty in the future value of stock $l$. In equilibrium, these factors just offset each other.

**Additional Frictions.** What drives our equivalence results is the fact that informed agents with common market information trade all assets, appropriately scaling down their trading intensities of stocks where the common information component is a lesser factor.

This suggests that to get around the equivalence result, and obtain the asymmetric cross-autocorrelation patterns, it suffices to introduce trading costs so that some agents with
common information do not trade assets with relatively less exposure to the common market innovation.

To illustrate this, we consider a simplified version of our model with pure market and idiosyncratic signals: Each period, $n_s$ agents see the idiosyncratic shock $g^s$ for asset $s$, and $n_m$ agents see the market shock $m$. If each agent can trade every asset, then Propositions 1 and 2 characterize the unique linear equilibrium and associated cross-auto covariances. However, suppose that due to some friction, some agents with market information do not trade all assets. In particular, suppose $n_{m,L}$ agents with market information trade the large stock and $n_{m,L} < n_{m,L}$ trade the small stock.

**Proposition 3** If $n_{m,L} > n_{m,L}$, price changes of stocks with relatively greater exposure to the common market shock have greater predictive power of cross-asset returns than price changes of stocks with less exposure

$$E[\Delta_{t+1} P^L \Delta_t P^L] < E[\Delta_{t+1} P^L \Delta_t P^L].$$

To motivate this result, consider the expected profit of traders with market information,

$$E[\Pi_s | m] = \frac{1}{\lambda_s} \left( \frac{\omega_s}{n_m + 1} \right)^2 n_m^2, \quad E[\Pi] = \frac{1}{\lambda_s} \left( \frac{\omega_s}{n_m + 1} \right)^2. \quad (5)$$

This expected profit is increasing in $\omega_s$ with a rate of $\omega_s^3$, and decreasing in $n_m$ with a rate of $n_m^{-3}$. Informally, if there are exogenous costs of trading that vary across traders, then the varying expected profitability of trading different stocks can induce variation in the numbers of agents speculating on the market component in each stock, i.e., $n_{m,L} > n_{m,L}$, and hence asymmetric cross-autocorrelations.

When more agents with information about the market innovation trade stocks with relatively more exposure to the market innovation, then changes in the prices of these stocks better predict price changes in stocks with less exposure to the market innovation, than the converse, retrieving the asymmetric cross-autocorrelation pattern. Most obviously, this is true when agents with market-wide information only trade a subset of stocks (e.g., large stocks).

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5 The two sets of informed traders (with market and with idiosyncratic information) may overlap without changing the results. This follows from the linear form of trading strategies.

6 This argument is only suggestive, as (formally) it gives rise to a mixture of normals, so that forecasts of market makers cease to be linear. However, the argument can be made rigorous if we restrict market makers to setting linear price schedules that earn zero profits *unconditionally*. Alternatively, we can modify the trade game such that informed traders with market information choose (commit to) the set of stocks they trade before observing their signals.
2 Conclusion

This paper explores patterns of lead-lag predictability across different portfolios in an asymmetric information asset pricing model with multiple assets. We first show that when market maker information is endogenized via endogenous informed trade, information heterogeneity across traders or assets cannot underlie the asymmetric patterns in the lead-lag predictability between various portfolios found in the data. We then obtain the asymmetric pattern by introducing trading frictions so that more agents with common information about market-wide asset values trade some stocks (e.g., large stocks), more than others (e.g., small stocks).

A Proofs

Proof of Proposition 1 We first show that if the price for stock \( s \in S \) is \( P_s^t = V^s_{t-1} + \lambda_s Z_s^t \), then the optimal trade of agent who sees signal \( \epsilon^s_t \) of stock \( s \) solves \( \lambda_s x^s_t, s' = k_{s',s} \epsilon^s_t \), where:

\[
\begin{align*}
    k_{s,s'} & = \frac{\sigma_s \psi_s}{\phi_s^2 + (1 + n_s) \psi_s} \chi(s = s') + \frac{\phi_s}{\phi_s^2 + (1 + n_s) \psi_s} K_s, \\
    K_s & = \frac{1}{1 + D} \left( \omega_s - \frac{n_s \sigma_s \psi_s \phi_s}{\phi_s^2 + (1 + n_s) \psi_s} \right), \\
    D & = \sum_{s''=1}^{S} \frac{n_{s''} \phi_{s''}^2}{\phi_{s''}^2 + (1 + n_{s''}) \psi_{s''}}.
\end{align*}
\]

The profit of a trader who sees a signal \( \epsilon^s_t \) and trades \( x^{s',s} \) in stocks \( s' = 1, ..., S \) is

\[
\pi^s = \sum_{s'=1}^{S} x^{s',s} (V^{s'} - P^{s'}) = \mathbb{E}_{[\pi^{s'} ; \epsilon^s ; \{x^{s',s}\}_{s'=1}^{S}]} \left[ V^{s'} | \epsilon^s, \{x^{s',s}\}_{s'=1}^{S} \right] - \mathbb{E}_{[\pi^{s'} ; \epsilon^s ; \{x^{s',s}\}_{s'=1}^{S}]} \left[ P^{s'} | \epsilon^s, \{x^{s',s}\}_{s'=1}^{S} \right],
\]

where

\[
\mathbb{E}_{[\pi^{s'} ; \epsilon^s ; \{x^{s',s}\}_{s'=1}^{S}]} \left[ V^{s'} | \epsilon^s \right] = V^s_{t-1} + \frac{\sigma_s \psi_s \chi(s = s') + \omega_s \phi_s}{\psi_s^2 + \phi_s^2} \epsilon^s_t.
\]

To compute the second expectation, first expand the pricing function,

\[
\mathbb{E}_{[\pi^{s'} ; \epsilon^s ; \{x^{s',s}\}_{s'=1}^{S}]} \left[ P^{s'} | \epsilon^s, \{x^{s',s}\}_{s'=1}^{S} \right] = V^s_{t-1} + \lambda_s \left( x^{s',s} + (n_s - 1) x^{s',s} + \sum_{s'' \neq s} \mathbb{E} \left[ x^{s',s''} | \epsilon^s, \{x^{s',s''}\}_{s''=1}^{S} \right] \right),
\]

and then use the linear form of the optimal trade of other agents,

\[
\mathbb{E}_{[\pi^{s'} ; \epsilon^s ; \{x^{s',s}\}_{s'=1}^{S}]} \left[ \lambda_s x^{s',s''} | \epsilon^s \right] = k_{s'',s} \mathbb{E}_{[\pi^{s''} ; \epsilon^s ; \{x^{s'',s''}\}_{s''=1}^{S}]} \left[ \epsilon^{s''} | \epsilon^s \right] = k_{s'',s} \frac{\phi_{s''} \phi_s}{\psi_s^2 + \phi_s^2} \epsilon^s_t,
\]

to obtain

\[
\mathbb{E}_{[\pi^{s'} ; \epsilon^s ; \{x^{s',s}\}_{s'=1}^{S}]} \left[ P^{s'} | \epsilon^s, \{x^{s',s}\}_{s'=1}^{S} \right] = V^s_{t-1} + \lambda_s \left( x^{s',s} + (n_s - 1) x^{s',s} \right) + \left( \sum_{s'' \neq s} n_{s''} \phi_{s''} \phi_s k_{s'',s} \right) \frac{\epsilon^s_t}{\psi_s^2 + \phi_s^2}.
\]
Substituting for $E[P_s' \mid \epsilon^s, \{x^{s',s}\}]$ into the profit of a speculator seeing signal $\epsilon^s_t$ yields

$$E[\pi^{s',s} \mid \epsilon^s, \{x^{s',s}\}]_{s' = 1} = \frac{\sigma_s \psi_s \chi(s' = s) + \omega_s \phi_s}{\psi^2_s + \phi^2_s} \epsilon^s_t \pi^{s',s}_t - \lambda^s_s \left( \pi^{s',s}_t \right)^2 - \lambda^s_s (n_s - 1) \bar{\pi}^{s',s}_t x^{s',s}_t - \left( \sum_{s'' \neq s} n_{s''} \phi_{s''} \phi_s k_{s''} \right) \epsilon^s_t \pi^{s',s}_t.$$  

Agents who see the same signals make the same trades, so the first-order conditions simplify:

$$\lambda^s_s (n_s + 1) x^{s',s}_t = \frac{\sigma_s \psi_s \chi(s' = s) + \omega_s \phi_s}{\psi^2_s + \phi^2_s} \epsilon^s_t - \left( \sum_{s'' \neq s} n_{s''} \phi_{s''} \phi_s k_{s''} \right) \epsilon^s_t \pi^{s',s}_t,$$

Hence,

$$(n_s + 1) k_{s,s'} = \frac{1}{\psi^2_s + \phi^2_s} \left[ \sigma_s \psi_s \chi(s' = s) + \omega_s \phi_s - \sum_{s'' = 1}^S n_{s''} \phi_{s''} \phi_s k_{s''} \right].$$

The above equation is implicit. To derive the explicit equation for $k_{s,s'}$, define:

$$K_{s'} \equiv \omega_{s'} - \sum_{s'' = 1}^S n_{s''} \phi_{s''} k_{s''} \Rightarrow \sum_{s'' \neq s} n_{s''} \phi_{s''} \phi_s k_{s''} = \omega_{s'} - K_{s'} - n_s \phi_s k_{s,s'},$$

and substitute it into the implicit equation for $k_{s,s'}$ to obtain:

$$(n_s + 1) (\psi^2_s + \phi^2_s) k_{s,s'} = \sigma_s \psi_s \chi(s' = s) + \phi_s K_s + n_s \phi^2_s k_{s,s'}.$$  

Therefore,

$$k_{s,s'} = \frac{\sigma_s \psi_s}{(n_s + 1) \psi^2_s + \phi^2_s} \chi(s' = s) + \frac{\phi_s}{(n_s + 1) \psi^2_s + \phi^2_s} K_{s'}.$$

This is the last line in equation (6). Substitute this back in the equation for $K_s$ to solve for

$$K_s = \omega_s - \sum_{s'' = 1}^S n_{s''} \phi_{s''} k_{s''},$$

$$= \omega_s - \sum_{s'' = 1}^S n_{s''} \phi_{s''} \left[ \frac{\sigma_{s''} \psi_{s''}}{(n_{s''} + 1) \psi^2_{s''} + \phi^2_{s''}} \chi(s'' = s) + \frac{\phi_{s''}}{(n_{s''} + 1) \psi^2_{s''} + \phi^2_{s''}} K_{s''} \right]$$

$$= \omega_s - \frac{n_s \sigma_s \psi_s \phi_s}{(n_s + 1) \psi^2_s + \phi^2_s} - \left( \sum_{s'' = 1}^S \frac{n_{s''} \phi^2_{s''}}{(n_{s''} + 1) \psi^2_{s''} + \phi^2_{s''}} \right) K_s.$$  

Solving this gives us the $K_s$ in equation (6).
We now show that the informed trading strategies imply that equilibrium pricing is linear with \( P_t^s = V_{t-1}^s + \lambda_s Z_t^s \), where:

\[
\lambda_s^2 \equiv \frac{1}{\eta_s^2} \left[ K_s(\omega_s - K_s) + n_s \sigma_s \psi_s k_{s,s} - \sum_{s'=1}^{S} (n_{s'} \psi_{s'} k_{s',s})^2 \right].
\] (8)

The zero profit condition of market makers implies that

\[
\mathbf{E}_{t-1} [(P_t^s - V_t^s) Z_t^s | Z_t^s] = 0 \quad \Rightarrow \quad P_t^s = \mathbf{E}_{t-1} [V_t^s | Z_t^s] = V_{t-1}^s + \mathbf{E}_{t-1} [\sigma_s g_t^s + \omega_s m_t | Z_t^s].
\] (9)

The joint normality of \((g_t^s, m_t, Z_t^s)\) implies that the pricing of stock \(s\) is characterized by

\[
\lambda_s = \frac{\text{cov}(\Delta_t V^s, Z_t^s)}{\text{var}(Z_t^s)} \quad \Rightarrow \quad \text{var}(\lambda_s Z_t^s) = \text{cov}(\lambda_s Z_t^s, \Delta_t V^s),
\]

where \(\Delta_t V^s = \sigma_s g_t^s + \omega_s m_t\). Further,

\[
\lambda_s Z_t^s = \lambda_s u^s + \sum_{s'=1}^{S} \lambda_s n_{s'} x^{s,s'} = \lambda_s u^s + \sum_{s'=1}^{S} n_{s'} k_{s',s} \psi^{s'}
\]

\[
= \lambda_s u^s + \sum_{s'=1}^{S} n_{s'} k_{s',s} \psi^{s'} + \left( \sum_{s'=1}^{S} n_{s'} \phi_{s'} k_{s',s} \right) m.
\] (10)

Hence,

\[
\text{var}(\lambda_s Z_t^s) = \lambda_s^2 \eta_s^2 + \sum_{s'=1}^{S} (n_{s'} k_{s',s} \psi^{s'})^2 + \left( \sum_{s'=1}^{S} n_{s'} \phi_{s'} \right)^2 \tag{11}
\]

\[
\text{cov}(\lambda_s Z_t^s, V_t^s) = n_s \psi_s \sigma_s k_{s,s} + \left( \sum_{s'=1}^{S} n_{s'} k_{s',s} \phi_{s'} \right) \omega_s. \tag{12}
\]

The pricing equation becomes:

\[
\lambda_s^2 \eta_s^2 + \sum_{s'=1}^{S} (n_{s'} k_{s',s} \psi^{s'})^2 + (\omega_s - K_s)^2 = n_s \psi_s \sigma_s k_{s,s} + (\omega_s - K_s) \omega_s.
\]

Solving this gives for \(\lambda_s\) gives equation \((8)\).

**Proof of Proposition 2** First observe that from equation \((8)\)

\[
\Delta_t P_t^s = \Delta_{t-1} V_t^s + \lambda_s Z_t^s - \lambda_s Z_{t-1}^s,
\]

\[
\Delta_{t-1} V_t^s = \sigma_s g_{t-1}^s + \omega_s m_{t-1}
\]

\[
\lambda_s Z_t^s = \lambda_s u^s + \sum_{s'=1}^{S} n_{s'} k_{s',s} \psi^{s'} + \left( \sum_{s'=1}^{S} n_{s'} \phi_{s'} k_{s',s} \right) m.
\]

7
Therefore:
\[
E\left[\Delta_{t+1} P^s \Delta_t P^{s'}\right] = E\left(\Delta_t V^s + \lambda_s Z_{t+1}^s - \lambda_s Z_t^s\right) \left(\Delta_{t-1} V^{s'} + \lambda_{s'} Z_{t-1}^{s'} - \lambda_{s'} Z_t^{s'}\right)
\]
\[
= E\left[\Delta_t V^s \lambda_{s'} Z_t^{s'}\right] - E\left[\lambda_s Z_t^s \lambda_{s'} Z_t^{s'}\right],
\]
which is zero for \(s = s'\) and when \(s \neq s'\) simplifies to:
\[
n_s \psi_s \sigma_s k_{s,s'} + \omega_s \sum_{s''=1}^m n_{s''} k_{s'', s'} \phi_{s''} - \sum_{s''=1}^m n_{s''}^2 \psi_{s''}^2 k_{s'', s'} k_{s''', s'''} - \left(\sum_{s''=1}^m n_{s''} \phi_{s''} k_{s'', s'}\right) \left(\sum_{s''=1}^m n_{s''} k_{s'', s'} \phi_{s''}\right)
\]
\[
\omega_{s'} - K_{s'}
\]
Simple algebra gives:
\[
E[\Delta_{t+1} P^s \Delta_t P^{s'}] = n_s \sigma_s \psi_s k_{s,s'} + K_s \omega_{s'} - K_s K_{s'} - \sum_{s''=1}^m (n_{s''} \psi_{s''})^2 k_{s'', s'} k_{s''', s'''} , \quad s' \neq s ,
\]
\[
= \frac{n_s \sigma_s \psi_s \phi_s}{\phi_s^2 + (n_s + 1) \psi_s^2} K_{s'} + \omega_{s'} K_s - K_s K_{s'} - \sum_{s''=1}^m (n_{s''} \psi_{s''})^2 k_{s'', s'} k_{s''', s'''}
\]
\[
= \omega_s K_{s'} + \omega_{s'} K_s - (2 + D) K_s K_{s'} - \sum_{s''=1}^m (n_{s''} \psi_{s''})^2 k_{s'', s'} k_{s''', s'''} ,
\]
which is symmetric. The last equality is obtained after using equation 6.

**Proof of Proposition 3** As a special case of Proposition 1 one can easily show that the unique linear equilibrium is characterized by
\[
x_t^s = \frac{\sigma_g t^s}{(n_s + 1) \lambda_s} , \quad y_t^s = \frac{\omega m_t}{(n_m + 1) \lambda_s} \quad \text{and} \quad \lambda_s^2 = \frac{n_s}{(n_s + 1)^2} \frac{\sigma_s^2}{\eta_s^2} + \frac{n_m}{(n_m + 1)^2} \frac{\omega_s^2}{\eta_s^2} .
\]
Finally, simple algebra shows that when the number of agents with market information that trade the high-beta stock are different from the number that trade the low-beta stock, \(n_{m,L} > n_{m,L}\), the cross-auto covariances are:
\[
E[\Delta_{t+1} P^L \Delta_t P^L] = n_{m,L} \left(\frac{\omega_L}{m_L + 1} \frac{\omega_L}{m_L + 1}\right) < n_{m,L} \left(\frac{\omega_L}{m_L + 1} \frac{\omega_L}{m_L + 1}\right) = E[\Delta_{t+1} P^L \Delta_t P^L]
\]
References


