Risks and Portfolio Decisions Involving Hedge Funds

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This article characterizes the systematic risk exposures of hedge funds using buy-and-hold and option-based strategies. Our results show that a large number of equity-oriented hedge fund strategies exhibit payoffs resembling a short position in a put option on the market index and therefore bear significant left-tail risk, risk that is ignored by the commonly used mean-variance framework. Using a mean-conditional value-at-risk framework, we demonstrate the extent to which the mean-variance framework underestimates the tail risk. Finally, working with the systematic risk exposures of hedge funds, we show that their recent performance appears significantly better than their long-run performance.

It is well accepted that the world of financial securities is a multifactor world consisting of different risk factors, each associated with its own factor risk premium, and that no single investment strategy can span the entire "risk factor space." Therefore investors wishing to earn risk premia associated with different risk factors need to employ different kinds of investment strategies. Sophisticated investors, like endowments and pension funds, seem to have recognized this fact as their portfolios consist

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of mutual funds as well as hedge funds.[1] Mutual funds typically employ a long-only buy-and-hold-type strategy on standard asset classes, and help capture risk premia associated with equity risk, interest rate risk, default risk, etc. However, they are not very helpful in capturing risk premia associated with dynamic trading strategies or spread-based strategies. This is where hedge funds come into the picture. Unlike mutual funds, hedge funds are not evaluated against a passive benchmark and therefore can follow more dynamic trading strategies. Moreover, they can take long as well as short positions in securities, and therefore can bet on capitalization spreads or value-growth spreads. As a result, hedge funds can offer exposure to risk factors that traditional long-only strategies cannot.[2]

As there is no "free lunch" in financial markets, questions arise regarding the kinds and nature of risks associated with different hedge fund strategies. This is a challenging task given the complex nature of the strategies and limited disclosure requirements faced by hedge funds. Out of a wide range of hedge fund strategies available in the marketplace, our knowledge to date is limited to the risks of two strategies: "trend following" analyzed by Fung and Hsieh (2001) and "risk arbitrage" studied by Mitchell and Pulvino (2001). Both studies find the risk return characteristics of the hedge fund strategies to be nonlinear and stress the importance of taking into account option-like features inherent while analyzing hedge funds.

We start with these insights and contribute to this emerging literature in several important ways. First, we extend our understanding of hedge fund risks to a wide range of equity-oriented hedge fund strategies. Instead of imposing a specific functional form, we allow for a flexible piecewise linear function of the market return to approximate the nonlinear payoffs of different hedge fund strategies. Our approach has the advantage that it is an operationally convenient method that can empirically characterize the risk of any generic hedge fund strategy. Second, we examine the implications of nonlinear option-like payoffs of hedge funds for portfolio decisions. We show how the conditional value-at-risk (CVaR) framework, which explicitly accounts for the negative tail risk, can be applied to

[1] For example, consider the investment strategies of large endowments like Harvard and Yale, or large pension funds like CALPERS and Ontario Teachers. We know from Fung and Hsieh (1997, 2001) that mutual funds predominantly employ relatively static trading strategies while hedge funds and Commodity Trading Advisers (CTAs) employ relatively dynamic trading strategies. Although they trade in similar asset classes as mutual funds, they show relatively low correlation with long-only type strategies.

[2] Although, in principle, investors can create exposure like hedge funds by trading on their own account, in practice they encounter many frictions due to incompleteness of markets like the publicly traded derivatives market and the financing market. Although the derivatives market for standardized contracts has grown a great deal in recent years, it is still very costly for an investor to create a customized payoff on individual securities. The same is true of the financing market as well, where investors encounter difficulties shorting securities and obtaining leverage. These frictions make it difficult for investors to create hedge fund-like payoffs by trading on their own accounts.

construct portfolios involving hedge funds.[3] We contrast our results with those obtained using the traditional mean-variance framework. Finally, we show how the limitation of a short history of hedge fund returns can be overcome by working with the underlying risk factors estimated through a multifactor model.[4] Since the underlying risk factors have a longer return history, this approach can provide insights into the long-term risk return trade-offs of hedge funds. On the whole, it provides important insights into the different hedge fund strategies, insights that are very helpful when making investment decisions like portfolio construction, risk management, benchmark design, manager compensation, etc., involving hedge funds.

It is well known that payoffs of managed portfolios will show option-like features [see Merton (1981) and Dybvig and Ross (1985)]. The importance of taking into account such option-like features, even when the fund manager does not have superior information and does not trade in derivatives, was first demonstrated by Jagannathan and Korajczyk (1986). The focus of this earlier stream of research was on assigning a value to the superior information that a skilled portfolio manager may possess by separating the skill into two dichotomous categories: market timing and security selection. Glosten and Jagannathan (1994) were the first to point out that even though it is rather difficult to separate a manager's ability clearly into two such categories, it is still possible to characterize the nature of the risk in managed portfolios and assign an overall value to the manager's skills by using derivative pricing methods. They suggested the inclusion of "excess returns on certain selected options on stock index portfolios as additional 'factor excess returns.' " Our article builds on this established theoretical framework supported by recent empirical evidence of option-like features in hedge fund payoffs.[5] Our use of exchange-traded options offers several advantages. First, they help capture the hedge fund risks in an intuitive manner. Second, being based on market prices, they embed investor preferences, information, and market conditions. Finally, being highly liquid and exchange traded, they enable replication of hedge fund payoffs.

We propose a two-step approach to characterize hedge fund risks. In the first step we estimate the risk exposures of hedge funds using a multifactor

[3] CVaR corresponds to the statistical mean of losses exceeding the VaR. While the VaR focuses only on the frequency of extreme events, CVaR focuses on both frequency and size of losses in case of extreme events.

[4] This is in the spirit of asset-based style factors proposed by Fung and Hsieh (2002a).

[5] Hedge funds provide an ideal testing ground for the application of Glosten and Jagannathan's (1994) approach for several reasons, some of which do not arise in case of mutual funds analyzed by them. This is because, unlike most mutual funds [see Koski and Pontiff (1999) and Almazan et al. (2001)], hedge funds frequently trade in derivatives. Further, hedge funds are known for their "opportunistic" nature of trading and a significant part of their returns arise from taking state-contingent bets.

model consisting of excess returns on standard assets and options on those assets as risk factors. In the second step we examine the ability of these risk factors to replicate the out-of-sample performance of hedge funds. Our out-of-sample analysis confirms that the risk factors estimated in the first step are not statistical artifacts of the data, but represent underlying economic risk exposures of hedge funds. Application of our approach at the hedge fund index level captures the "popular bets" taken (i.e., common risks borne) by a large number of hedge funds that were operating during the sample period, while application at the individual hedge fund level provides information about the systematic risks borne by that specific hedge fund.

Hedge funds may exhibit nonnormal payoffs for various reasons such as their use of options, or option-like dynamic trading strategies or strategies that lose money during market downturns. For example, during the Russian debt crisis in August 1998 a wide range of hedge funds reported large losses. This suggests that hedge funds may be bearing significant left-tail risk. Regulatory bodies such as the Basle committee have recognized this feature and have emphasized the importance of tail risk and the use of risk management frameworks such as the Value-at-Risk (VaR). Keeping this in mind, we employ a mean-conditional Value-at-Risk (M-CVaR) framework for portfolio construction involving hedge funds. Using this framework, we examine the extent to which the traditional mean-variance framework underestimates the tail risk of hedge funds.

We address the common problem of the short history of hedge fund returns one encounters while conducting empirical research on hedge funds. Since most hedge fund databases report their returns from the early 1990s, a natural question arises as to how the hedge funds would have performed during extreme events in the past, such as the Great Depression of the 1930s, the oil shock of the early 1970s, or the stock market crash of 1987. We shed light on this issue by working with the underlying risk factors that have a longer return history. Assuming that the hedge funds were bearing the same systematic risk exposures as those during the 1990s, we estimate their returns prior to our sample period and compare their long-term performance with their performance during the 1990s. We show how this approach can help investors get a long-term perspective on the risk return trade-offs of hedge funds.

Our analysis provides three main findings. First, we find that the nonlinear option-like payoffs are not restricted only to "trend followers" and "risk arbitrageurs," but are an integral feature of the payoffs on a wide range of hedge fund strategies. In particular, we observe that the payoffs on a large number of equity-oriented hedge fund strategies resemble those from writing a put option on the equity index. Second, we find that the expected tail losses of mean-variance optimal portfolios can be underestimated by as high as 54% compared with M-CVaR optimal portfolios.

This suggests that ignoring the tail risk of hedge funds can result in significantly higher losses during large market downturns. Finally, our analysis using extrapolated hedge fund returns during the 1927–1989 period suggests that their performance during the last decade is not representative of their long-term performance. In particular, we find that the expected losses beyond VaR during the 1927–1989 period can be about twice of those during the 1990s. We also find that their mean returns during the 1927–1989 period are significantly lower and their standard deviations are significantly higher compared to those of their recent performance. These findings have important implications for risk management and portfolio decisions involving hedge funds. They also provide support to the theoretical modeling of hedge funds in the Kyle and Xiong (2001) framework.

The rest of the article is organized as follows. Section 1 provides the theoretical framework. Section 2 contains the description of data and the risk factors (buy-and-hold and option-based) used in our multifactor model. Section 3 presents the model, the in-sample analysis, and various robustness checks, while Section 4 conducts the out-of-sample analysis. Section 5 develops the M-CVaR framework and contrasts the findings with the traditional mean-variance framework. Section 6 examines the long-term performance of hedge funds and compares it with their recent performance. Section 7 offers concluding remarks and suggestions for future research.

1. Theoretical Framework

Linear factor models such as the capital asset pricing model (CAPM) and the arbitrage pricing theory (APT) have been the foundation of most of the theoretical and empirical asset pricing literature. Unfortunately these theories constrain the relation between risk factors and returns to be linear. Therefore they cannot price securities whose payoffs are nonlinear functions of the risk factors. Researchers have addressed this problem using a nonlinear asset pricing framework [see, e.g., Rubinstein (1973), Kraus and Litzenberger (1976), Dybvig and Ingersoll (1982), Bansal and Viswanathan (1993), and Bansal, Hsieh, and Viswanathan (1993)]. More recently, while investigating the importance of nonlinearities arising from conditional skewness, Harvey and Siddique (2000a, 2000b) specify the marginal rate of substitution to be quadratic in the market return, namely,

$$m_{t+1} = a_t + b_t R_{M,t+1} + c_t R^2_{M,\,t+1}, \tag{1}$$

and derive an asset pricing model of the following form:

$$E_t(r_{i,t+1}) = A_t E_t(r_{M,t+1}) + B_t E_t(R^2_{M,t+1}). \tag{2}$$

The aim of these studies is to price securities with asymmetric nonlinear payoffs. However, there exists another strand of literature that is related to the nonlinear payoffs, but which focuses on the use of options to characterize the nonlinearities [Breeden and Litzenberger (1978)] and assign a value to the nonlinearities. In particular, Glosten and Jagannathan (1994) show how a value can be assigned to the skill of the manager generating a nonlinear payoff. More importantly, they show that for valuation purposes it is not necessary to replicate the nonlinear payoff by a collection of options, but it is only necessary to replicate that part of the payoff that has nonzero value. For this purpose, it is only necessary to approximate the nonlinear payoff by a collection of options on a selected number of benchmark index returns. There will be some residual risk, but that residual risk will not be priced.

Glosten and Jagannathan (1994) use the contingent claim-based specification of the form

$$\begin{aligned} R_p = \alpha + \beta_1 R_m + \beta_2 \max(R_m - k_1, 0) + \beta_3 \max(R_m - k_2, 0) \\ + \beta_4 \max(R_m - k_3, 0) + \varepsilon. \end{aligned} \tag{3}$$

In this article we build on Glosten and Jagannathan's (1994) framework and specify a flexible piecewise linear form involving call and put options on the market index, namely,

$$\begin{aligned} R_p = \alpha + \beta_1 R_m + \beta_2 \max(R_m - k_1, 0) + \beta_3 \max(R_m - k_2, 0) \\ + \beta_4 \max(k_1 - R_m, 0) + \beta_5 \max(k_3 - R_m, 0) + \varepsilon. \end{aligned} \tag{4}$$

Since the payoffs on options can be expressed as a polynomial function of the market return, our option-based specification is related to the earlier stream of literature expressing the pricing kernel as a polynomial function of market return.[6] In terms of implementation, our augmentation of the linear beta model with nonlinear option-based factors (which have skewed payoffs) is similar in spirit to Harvey and Siddique's (2000a) augmentation of Fama and French's (1993) three-factor model by a nonlinear factor derived from skewness (i.e., the mimicking return on the high minus low coskewness portfolio). The main motivation behind our use of options is to have a liquid and frequently traded asset whose payoff relates in a nonlinear way with the market return and whose market prices can be used to compute returns to such payoffs.

Having described the theoretical framework and how our model relates to other nonlinear models used in the literature to price securities with nonlinear payoffs, and to assign a value to the skill of a manager

[6] However, it is important to note that specifying the marginal rate of substitution to be quadratic in market return, as in Equation (1), is different from it being related to the payoffs on put and call options on the market.

generating a nonlinear payoff, we proceed to the description of data and risk factors used in our multifactor model.

2. Description of Data and Risk Factors

In this article we analyze equity-oriented hedge fund strategies. The reason for focusing on these strategies is the availability of high-quality data on exchange-traded options on broad-based equity indexes such as Standard and Poors' (S&P) 500 composite index. We analyze six hedge fund strategies whose payoff arises primarily from relative mispricings of securities rather than the movement of the market as a whole, namely event arbitrage, restructuring, event driven, relative value arbitrage, convertible arbitrage, and equity hedge (long/short equity). We also investigate two hedge fund strategies whose payoff arises primarily from taking directional bets, namely equity non-hedge and short selling (dedicated short-bias). It is well known that hedge fund indexes differ from each other in the way they are constructed.[7] Further, they may be subject to different levels of survivorship and backfilling biases (Fung and Hsieh [2002b]. Survivorship bias arises due to exclusion of funds from the database that die during the sample period, while backfilling or "instant history" bias arises when the database backfills the historical return data of a fund before its entry into the database. The former is around 3% per annum while the latter is around 1.4% per annum [see Brown, Goetzmann, and Ibbotson (1999) and Fung and Hsieh (2000b, 2002b)]. Therefore, for the sake of robustness, we conduct our analysis using both the Hedge Fund Research (HFR) and CSFB/Tremont indexes.

From the HFR indexes, we select event arbitrage, restructuring, event driven, relative value arbitrage, convertible arbitrage, equity hedge, equity non-hedge, and short selling indexes. We also select four CSFB/Tremont indexes, namely event driven, convertible arbitrage, long/short equity, and dedicated short-bias that correspond to event driven, convertible arbitrage, equity hedge, and short selling HFR indexes. Our sample consists of monthly returns on the HFR indexes from January 1990 to June 2000 and on the CSFB/Tremont indexes from January 1994 to June 2000. We validate our findings of economic risk exposures of hedge funds using out-of-sample data from July 2000 to December 2001.[8]

Our multifactor model uses a set of buy-and-hold and option-based risk factors. The buy-and-hold risk factors consist of indexes representing

[7] The Hedge Fund Research (HFR) indexes are equally weighted and therefore give relatively more weight to the performance of smaller hedge funds while the Credit Suisse First Boston (CSFB)/Tremont indexes are value weighted (i.e., weighted by assets under management) and hence give relatively more weight to the performance of larger hedge funds. See www.hfr.com and www.hedgeindex.com for the index construction details.

[8] We thank the referee for suggesting this approach.

equities (Russell 3000 index, lagged Russell 3000 index,[9] Morgan Stanley Capital International (MSCI) world excluding the USA index, and MSCI emerging markets index), bonds (Salomon Brothers government and corporate bond index, Salomon Brothers world government bond index, and Lehman high yield index), Federal Reserve Bank competitiveness-weighted dollar index, and the Goldman Sachs commodity index. We also include three zero-investment strategies representing Fama and French's (1993) "size" factor (small minus big, or SMB), "book--to-market" factor (high minus low, or HML), and Carhart's (1997) "momentum" factor (winners minus losers). Finally, to capture credit risk, we include the change in the default spread (the difference between the yield on the BAA-rated corporate bonds and the 10-year Treasury bonds) as an additional factor.

Our option-based risk factors consist of highly liquid at-the-money (ATM) and out-of-the-money (OTM) European call and put options on the S&P 500 composite index trading on the Chicago Mercantile Exchange. Our use of options with different degrees of moneyness allows a flexible piecewise linear risk-return relation. The process of buying an ATM call option on the S&P 500 index works as follows. On the first trading day in January, buy an ATM call option on the S&P 500 index that expires in February. On the first trading day in February, sell the option bought a month ago (i.e., at the beginning of January) and buy another ATM call option on the S&P 500 index that expires in March. Repeating this trading pattern every month provides the time series of returns on buying an ATM call option. A similar procedure provides the time series of returns on buying OTM call options.[10] We select the ATM option as the one whose present value of the strike price is closest to the current index value. We select the OTM call (put) option to be the one with the next higher (lower) strike price.[11] We denote the ATM call (put) option on the S&P 500 index by SPC_a (SPP_a) and the OTM call (put) option by SPC_o (SPP_o). Using price data from the Institute for Financial Markets, we compute monthly returns to these option-based risk factors.

Our approach has the flexibility to combine long and/or short positions in calls and/or puts with differing strike prices without having to prespecify whether it is a long or a short position, the number of units of each

[9] The use of the lagged Russell 3000 index accounts for the effect of nonsynchronous trading and was suggested by Asness, Krail, and Liew (2001).

[10] We do not consider in-the-money (ITM) options, as their payoffs can be replicated by a combination of underlying assets and risk-free assets along with an OTM option. For example, the maturity payoff on an ITM call option can be replicated by a long position in the underlying asset, a long position in the risk-free asset, and a long position in an OTM put with the same strike price.

[11] Options are available in strike price increments of five index points. On average, the ratio of index price to present value of the strike price for our ATM options is 1.00 while that for our OTM call (put) options is 0.99 (1.01). We discuss the robustness for our results for specifying higher degrees of out-of-the-moneyness in Section 3.2.

option, and the strike price of each option. It is this flexibility that enables our option-based risk factors to effectively capture the nonlinear payoffs of hedge funds.

We report the summary statistics for the HFR indexes and our buy-and-hold and option-based risk factors during the January 1990 to June 2000 period in panels A and B of Table 1. We also provide the summary statistics for the CSFB/Tremont indexes during the January 1994 to June 2000 period in panel C of Table 1. We show the correlations between the different hedge fund indexes and the risk factors in Table 2. As can be seen, all HFR indexes and three out of four CSFB/Tremont indexes show significant correlation with the Russell 3000 index. A large number of hedge fund indexes also show significant correlation with Fama and French's (1993) size factor. Mitchell and Pulvino (2001) find that the risk arbitrage strategy shows zero correlation with the market during up-market conditions, but large positive correlation during down-market conditions. In order to examine whether this is true for a wide range of hedge fund indexes, we use a regression specification that allows for separate intercept and slope coefficients when the market index is above and below its median return. We report our findings in Table 3. We find that a large number of hedge fund indexes show no correlation in up-market conditions, but a positive correlation in down-market conditions. This asymmetry of betas or factor loadings in up-market versus down-market conditions confirms the nonlinear nature of hedge fund payoffs. It also suggests that the extent of diversification benefits offered by hedge funds would be smaller during down-market conditions.

3. Multifactor Model and Results

As discussed in the introduction, we employ a two-step procedure to characterize the systematic risk exposures of hedge funds. The first step involves identifying statistically significant factors that ex post capture in-sample variation in hedge fund returns. Toward that end, we regress the net-of-fee monthly excess return (in excess of the risk-free rate of interest) on a hedge fund index on the excess return on buy-and-hold and option-based risk factors in a multifactor framework.[12] In particular, we estimate the following regression

$$R_t^i = c^i + \sum_{k=1}^{K} \lambda_k^i F_{kt} + u_t^i, \tag{5}$$

where R_t^i is the net-of-fees excess returns (in excess of the risk-free rate) on hedge fund index i during month t, c^i is the intercept for hedge fund index i

[12] As returns on option-based strategies have a larger order of magnitude compared to the buy-and-hold strategies, we scale them by a factor of 100 and use the scaled returns in our multifactor model.

Table 1
Summary statistics

Panel A: HFR hedge fund indexes

Hedge fund strategy	Mean	SD	Median	Skew	Kurtosis	Minimum	Maximum
Non-Directional							
Event arbitrage	1.03	1.32	1.33	−3.24	17.18	−6.46	2.90
Restructuring	1.29	1.90	1.35	−0.81	8.88	−8.50	7.06
Event driven	1.33	1.94	1.53	−1.62	9.42	−8.90	5.13
Relative value arbitrage	1.15	1.16	1.29	−1.26	13.31	−5.80	5.72
Convertible arbitrage	0.95	1.01	1.16	−1.48	6.30	−3.19	3.33
Equity hedge	1.82	2.65	1.82	0.10	4.57	−7.65	10.88
Directional							
Equity non-hedge	1.71	4.06	2.28	−0.59	4.17	−13.34	10.74
Short selling	0.07	6.40	−0.16	0.13	4.64	−21.21	22.84

Panel B: Risk factors

Risk factor	Mean	SD	Median	Skew	Kurtosis	Minimum	Maximum
			Buy-and-hold risk factors				
Equity							
Russell 3000	1.39	3.94	1.69	−0.67	4.75	−15.32	11.22
MSCI world excluding U.S.	0.66	4.83	0.71	−0.18	3.49	−13.47	14.67
MSCI emerging markets	1.01	6.80	1.41	−0.64	5.49	−28.91	16.53
Fama and French SMB factor	−0.03	3.46	−0.08	0.54	6.15	−11.66	15.40
Fama and French HML factor	−0.31	4.16	−0.43	−1.14	9.73	−21.51	14.23
Momentum factor	0.94	4.18	1.17	−0.27	4.75	−11.47	13.77
Bond							
SB government and corporate bond	0.63	1.25	0.77	−0.06	3.25	−2.37	4.65
SB world government bond	0.63	1.81	0.75	0.16	3.39	−3.63	6.11
Lehman high yield	−0.10	3.16	0.05	−4.16	35.60	−25.47	10.16
Default spread	−0.09	1.65	−0.21	0.06	3.36	−5.50	3.67
Currency							
FRB competitiveness-weighted dollar	0.45	1.20	0.30	0.42	3.68	−2.78	3.96
Commodity							
Goldman Sachs commodity	0.65	5.04	0.79	0.54	4.36	−12.28	18.52
			Option-based risk factors				
S&P 500 at-the-money call	4.77	84.09	−17.01	0.76	2.80	−98.57	236.24
S&P 500 out-of-the-money call	3.36	93.80	−23.69	1.04	3.53	−99.35	300.60
S&P 500 at-the-money put	−24.38	84.72	−57.04	2.20	8.77	−95.30	386.02
S&P 500 out-of-the-money put	−27.30	91.49	−62.76	2.69	11.67	−95.80	422.34

Panel C: CSFB/Tremont hedge fund indexes

Hedge fund strategy	Mean	SD	Median	Skew	Kurtosis	Minimum	Maximum
Nondirectional							
Event driven	1.00	1.97	1.26	−3.59	24.01	−11.77	3.68
Convertible arbitrage	0.83	1.50	1.15	−1.59	6.62	−4.68	3.57
Long/short equity	1.41	3.68	1.36	−0.04	5.16	−11.43	13.01
Directional							
Dedicated short-bias	−0.26	5.26	−0.39	1.11	6.18	−8.69	22.71

This table shows the means, standard deviations (SD), medians, skewness (Skew), kurtosis, and minimum and maximum of returns for 8 HFR hedge fund indexes (panel A), 12 buy-and-hold, and 4 option-based risk factors (panel B) during January 1990 to June 2000 and 4 CSFB/Tremont hedge fund indexes (panel C) during January 1994 to June 2000.

Table 2
Correlation between the hedge fund indexes and asset class factors

	HFR								CFSB/TREMONT			
	EA	REST	ED	RVAL	CA	EH	ENH	SHORT	ED	CA	EH	SHORT
RUS	**0.49**	**0.42**	**0.66**	**0.39**	**0.39**	**0.67**	**0.81**	**−0.71**	**0.61**	0.18	**0.68**	**−0.67**
MXUS	0.29	0.29	**0.43**	0.30	0.27	**0.45**	**0.52**	**−0.49**	**0.61**	0.13	**0.66**	**−0.64**
MEM	**0.36**	**0.54**	**0.58**	0.41	0.39	0.54	**0.63**	**−0.53**	**0.63**	0.23	**0.65**	**−0.61**
SMB	0.29	**0.48**	**0.49**	0.38	0.30	0.56	**0.57**	**−0.57**	**0.45**	0.20	**0.54**	**−0.49**
HML	−0.13	−0.12	−0.29	−0.05	−0.16	−0.59	**−0.57**	**0.68**	**−0.53**	−0.06	**−0.72**	**0.72**
MOM	−0.04	−0.22	−0.03	**−0.35**	−0.18	0.16	0.07	−0.14	0.12	−0.14	0.28	−0.18
SBG	0.14	0.05	0.15	0.04	0.20	0.15	0.17	−0.11	0.05	0.12	0.13	−0.06
SBW	−0.03	−0.20	−0.10	−0.15	−0.05	0.00	0.01	−0.05	−0.11	−0.27	0.00	0.04
LHY	0.28	**0.49**	**0.39**	0.32	0.32	0.28	**0.42**	−0.30	**0.48**	**0.45**	**0.46**	**−0.40**
DEFSPR	−0.18	−0.21	−0.26	−0.15	−0.25	−0.21	−0.26	0.18	−0.15	−0.17	−0.21	0.10
FRBI	0.01	0.19	0.06	−0.01	−0.12	−0.06	−0.05	0.10	−0.12	−0.01	−0.24	0.27
GSCI	−0.08	0.04	0.03	0.07	0.05	0.13	−0.05	0.03	0.18	0.12	0.19	−0.12

This table shows the correlations between the 8 HFR hedge fund indexes and the 12 buy-and-hold risk factors during our sample period (January 1990 to June 2000). The table also shows the correlation between the 4 CSFB/Tremont hedge fund indexes and the 12 risk factors during the entire sample period from January 1994 and June 2000. The buy-and-hold risk factors are the Russell 3000 index (RUS), MSCI excluding the U.S. index (MXUS), MSCI emerging markets index (MEM), Fama and French size and book-to-market factors (SMB and HML), momentum factor (MOM), Salomon Brothers government and corporate bond index (SBG), Salomon Brothers world government bond index (SBW), Lehman high yield composite index (LHY), Federal Reserve Bank competitiveness-weighted dollar index (FRBI), Goldman Sachs commodity index (GSCI), and the change in the default spread in basis points (DEFSPR). The abbreviations for different hedge fund strategies are event arbitrage (EA), restructuring (REST), event driven (ED), relative value arbitrage (RVAL), convertible arbitrage (CA), equity hedge or long/short equity (EH), equity non-hedge (ENH), and short selling or dedicated short-bias (SHORT). Correlations significant at the Bonferroni-adjusted significance level of 5% are shown in bold type.

Table 3
Correlation between the hedge fund indexes and risk factors during different market conditions

	HFR								CFSB/TREMONT			
	EA	REST	ED	RVAL	CA	EH	ENH	SHORT	ED	CA	EH	SHORT
α_0	**1.24**	**2.51**	**2.40**	**1.88**	**1.22**	**2.27**	**1.94**	0.50	**1.88**	1.89	1.92	0.12
β_0	0.03	−0.18	−0.04	−0.10	0.00	0.21	0.51	**−0.90**	−0.02	−0.18	0.31	**−0.76**
β_1	0.01	−0.88	−1.08	−0.63	−0.28	−0.98	−1.04	1.10	**−0.89**	−1.07	−1.45	1.05
γ	**0.31**	**0.67**	**0.58**	**0.35**	**0.16**	**0.32**	**0.52**	−0.34	**0.55**	0.31	0.42	−0.46
Adj. R^2	33.45	34.93	55.15	26.53	17.17	45.33	67.16	49.75	51.48	4.10	48.84	65.89

This table shows the results of the following regressions for eight HFR and four CSFB/Tremont hedge fund indexes during January 1990 to June 2000 for HFR and January 1994 to June 2000 for CSFB/Tremont:

$$R_t^i = \alpha_0^i + \beta_0^i \mathrm{RUS}_t + \beta_1^i D + \gamma^i D \times \mathrm{RUS}_t + \varepsilon_t^i,$$

where R_t^i is the returns on hedge fund index i during month t, α_0^i is the intercept for hedge fund index i, β_0^i is the slope coefficient on the Russell 3000 index, β_1^i is the slope coefficient on the dummy variable D ($D=1$ if the return for the Russell 3000 index is less than its median return and $D=0$ if the return for the Russell 3000 index is equal to or more than the median return), γ^i is the slope coefficient on the interaction terms $D \times RUS_t$, and ε_t^i is the error term. Various hedge fund strategies are event arbitrage (EA), restructuring (REST), event driven (ED), relative value arbitrage (RVAL), convertible arbitrage (CA), equity hedge or long/short equity (EH), equity non-hedge (ENH), and short selling or dedicated short-bias (SHORT). Parameters significantly different from zero at the 5% level are shown in bold type.

over the regression period, λ_k^i is the average factor loading of hedge fund index i on the k-th factor during the regression period, F_{kt} is the excess return on the k-th factor during month t, $(k=1,\ldots,K)$ where the factor could be a buy-and-hold or an option-based risk factor, and u_t^i is an error term.

Given the lack of transparency and the large number of possible market and trading strategy combinations the hedge funds can follow, it is a challenging task to identify the dominant risk factors using limited data on their returns. This problem has been well recognized in the hedge fund literature. Researchers have addressed this problem by using a stepwise regression procedure either explicitly [Liang (1999), Fung and Hsieh (2000a)] or implicitly [Fung and Hsieh (2001, Table 5)] while identifying significant risk factors. The stepwise regression involves adding and/or deleting variables sequentially depending on the F-value. One of the benefits of this procedure lies in its parsimonious selection of factors, while one of its shortcomings lies in the breakdown of standard statistical inference. The latter is a potential concern; however, it should only worsen the ability of the parsimoniously extracted factors to explain out-of-sample variation in hedge fund returns. Given that we obtain within-sample results that are consistent with other researchers and that we are able to replicate the out-of-sample performance of hedge funds, we believe that the benefits of using a stepwise regression procedure outweigh its limitations.

3.1 Common risk exposures of hedge funds belonging to the HFR indexes

We describe in Table 4 the factors that exhibit a statistically significant relation in our stepwise regression procedure when the dependent variable is the returns on HFR's event driven, event arbitrage, restructuring, relative value arbitrage, convertible arbitrage, equity hedge, equity non-hedge, and short selling indexes.[13]

3.1.1 Significant risk exposures of HFR event arbitrage index. We find a nonlinear risk-return trade-off with the event arbitrage index showing significant factor loading on the risk factor corresponding to writing at the OTM put option on the S&P 500 index (SPP_o). This result is intuitive, as event arbitrage strategy involves the risk of deal failure. A larger fraction of deals fail when markets are down and the event arbitrage strategy incurs losses. In contrast, when markets are up, a larger proportion of deals go through and the strategy makes profits. But the profits are

[13] We specify a 5% significance level for including an additional variable in our stepwise regression procedure. Tables 4 and 5 report the significant factors and the adjusted R^2. We determine the significance using heteroscedasticity and autocorrelation-consistent standard errors.

Table 4
Results with HFR equally-weighted indexes

Event arbitrage		Restructuring		Event driven		Relative value arbitrage		Convertible arbitrage		Equity hedge		Equity non-hedge		Short selling	
Factors	λ	Factors	λ	Factors	λ	Factors	λ	Factors	λ	Factors	λ	Factors	λ	Factors	λ
C	0.04	*C*	0.43	*C*	0.20	*C*	0.38	*C*	0.24	*C*	0.99	*C*	0.56	*C*	−0.07
SPP_o	−0.92	SPP_o	−0.63	SPP_o	−0.94	SPP_o	−0.64	SPP_a	−0.27	RUS	0.41	RUS	0.75	SPC_o	−1.38
SMB	0.15	SMB	0.24	SMB	0.31	MOM	−0.08	LRUS	0.10	SMB	0.33	SMB	0.58	RUS	−0.69
HML	0.08	HML	0.12	HML	0.12	SMB	0.17	SMB	0.05	HML	−0.08	MEM	0.05	SMB	−0.77
		LRUS	0.06	RUS	0.17	HML	0.08	MEM	0.03	GSCI	0.08			HML	0.40
		LHY	0.13	MEM	0.06	MXUS	0.04	SBG	0.16						
		FRBI	0.27												
		MEM	0.09												
Adj-R^2	44.04	Adj-R^2	65.57	Adj-R^2	73.38	Adj-R^2	52.17	Adj-R^2	40.51	Adj-R^2	72.53	Adj-R^2	91.63	Adj-R^2	82.02

This table shows the results of the regression $R_t^i = c^i + \sum_{k=1}^{K} \lambda_k^i F_{k,t} + u_t^i$ for the eight HFR indexes during the full sample period from January 1990 to June 2000. The table shows the intercept (*C*), statistically significant (at the 5% level) slope coefficients on the various buy-and-hold and option-based risk factors and adjusted R^2 (Adj-R^2). The buy-and-hold risk factors are Russell 3000 index (RUS), lagged Russell 3000 index (LRUS)), MSCI excluding the U.S. index (MXUS), MSCI emerging markets index (MEM), Fama and French size and book-to-market factors (SMB and HML), momentum factor (MOM), Salomon Brothers government and corporate bond index (SBG), Salomon Brothers world government bond index (SBW), Lehman high yield composite index (LHY), Federal Reserve Bank competitiveness-weighted dollar index (FRBI), Goldman Sachs commodity index (GSCI), and the change in the default spread in basis points (DEFSPR). The option-based risk factors include the at-the-money and out-of-the-money call and put options on the S&P 500 composite index ($SPC_{a/o}$ and $SPP_{a/o}$). For the two call and put option-based strategies, subscripts *a* and *o* refer to at-the-money and out-of-the-money, respectively.

unrelated to the extent to which the market goes up. Thus the payoff to the event arbitrage strategy resembles that obtained by writing a naked put option on the market.

Fama and French's size (SMB) factor shows a significant relation, suggesting that returns to event arbitrage strategies resemble those achieved by going long on small stocks and short on large stocks. This is intuitive as well, since the size of the target firm is generally smaller than that of the acquiring firm. Going long the target's stock and short the acquirer's stock naturally results in a long exposure on Fama and French's size factor. Fama and French's value (HML) factor also shows a significant relation, suggesting a tilt toward value stocks. This would happen if the hedge funds were following an event arbitrage strategy and the growth firms were trying to acquire value firms.

It is interesting to compare and contrast our analysis of the risks of an event arbitrage strategy with Mitchell and Pulvino's (2001) findings of the risks of the same strategy. They select 4750 merger events from 1963 to 1998 and examine the risks in a stock merger (by going long the target's stock and going short the acquirer's stock) and those in a cash merger (by going long the target's stock). They find that the risk of merger or event arbitrage strategy resembles that of writing a naked put option on the market and having a long exposure to Fama and French's size (SMB) factor. Of interest is that our multifactor model also selects writing a put option on the S&P 500 index and going long on Fama and French's size factor as dominant risk factors. These striking similarities suggest that our approach is able to capture dominant risk exposures of hedge funds following the event arbitrage strategy.

3.1.2 Significant risk exposures of HFR restructuring index. Restructuring strategy involves investing in the securities of firms in financial distress (i.e., firms that have filed for Chapter 11 or are undergoing some form of reorganization). For this strategy, similar to the event arbitrage index, we find a nonlinear risk-return trade-off. In particular, it shows a significant factor loading on risk factor corresponding to writing at an OTM put option on the S&P 500 index (SPP_o). This result is intuitive as the probability of firms emerging from financial distress is lower when the markets are down due to firms losing business during market downturns. Thus the payoff to this strategy resembles that obtained by writing a put option on the market.

In addition, we find Fama and French's size (SMB) factor showing a significant relation with the restructuring index. This is not surprising because smaller firms are more likely to be in distress. Further, we find that the Fama and French value (HML) factor also shows a significant relation. This is again consistent with high book-to-market ratio firms being more likely to be in distress.

Typically these securities are illiquid and infrequently traded. Our finding of a significant factor loading on the lagged Russell 3000 index and Lehman high yield index is consistent with this notion. The restructuring index also shows a significant factor loading on the FRB competitiveness-weighted dollar index and MSCI emerging market index. This may be due to the managers investing in distressed firms from emerging markets or those exposed to emerging markets.

3.1.3 Significant risk exposures of the HFR event driven index. Similar to the event arbitrage and restructuring indexes, we find a nonlinear risk-return trade-off in the case of the event driven index. This is manifested through a short position in an OTM put option on the S&P 500 index (SPP_o). Event driven strategy involves taking bets on events such as mergers, takeovers, and reorganizations. The risk in this strategy pertains to the nonrealization of such events. This is more likely to happen during market downturns. The short position in the put option is consistent with this economic interpretation.

We also find a positive loading on Fama and French's size (SMB) and value (HML) factors, Russell 3000, and lagged Russell 3000 indexes. As event driven strategy is similar to event arbitrage and restructuring strategies, we find the risk factors to be similar and existing for similar reasons as mentioned before.

3.1.4 Significant risk exposures of the HFR relative value arbitrage index. The relative value arbitrage strategy attempts to take advantage of relative pricing discrepancies between instruments such as equities, debt, and derivative securities. As in the previous cases, we find that it also exhibits a nonlinear risk-return relation with the equity market index. The relative value arbitrage index payoff resembles that from a short position in an OTM put option on the S&P 500 index (SPP_o) suggesting that these strategies lose money during large down moves in the equity market. Carhart's (1997) momentum factor is also significant with a negative factor loading, suggesting that relative value arbitrage funds follow a "contrarian" strategy. This finding is intuitive. Hedge funds employing such strategies follow securities with similar fundamental value and, when their prices diverge, they buy undervalued securities (losers) and sell overvalued securities (winners). This is opposite of what the momentum traders do, namely buy winners and sell losers. As before, we also find Fama and French's size (SMB) and value (HML) factors to be significant. This finding is consistent with the results of Gatev, Goetzmann, and Rouwenhorst (1999), who replicate returns of pairs trading strategy, which is one of the strategies followed by relative value arbitrage funds.

3.1.5 Significant risk exposures of the HFR convertible arbitrage index. Convertible arbitrage strategy attempts to take advantage of relative pricing discrepancies between the theoretical and market prices of convertible bonds. If a convertible bond appears to be undervalued, then the manager may purchase the bond and hedge out some of the risk components such as equity risk, credit risk, and interest rate risk. As in the previous cases, we find that it also exhibits a nonlinear risk-return relation with the equity market index. The convertible arbitrage index payoff resembles that from a short position in an ATM put option on the S&P 500 index (SPP_a), suggesting that these strategies lose money during large down moves in the equity market. The lagged Russell index is also significant, suggesting the illiquid and infrequent trading nature of the bonds. Similar to the restructuring and event driven indexes, we find that the convertible arbitrage index also shows significant loading on Fama and French's size (SMB) index and the MSCI emerging market index.

3.1.6 Significant risk exposures of the HFR equity hedge and equity non-hedge indexes. The HFR equity hedge index covers the original long--short strategy followed by Albert Winslow Jones in 1949. HFR includes funds that follow long-short strategies into equity hedge and equity non-hedge categories. Hedge funds that aim to have relatively low net long exposure are included in the HFR equity hedge index, while those with relatively high net long exposure are included in the HFR equity non-hedge index. This is confirmed by their betas with respect to the Russell 3000 index with the equity hedge (equity non-hedge) index showing a beta of 0.41 (0.75). Both the indexes show long exposure to Fama and French's size (SMB) factor. This finding is intuitive, as one would expect the small stock universe to be less researched and therefore one has a higher probability of finding mispriced stocks. A long exposure to the SMB factor suggests that these managers buy undervalued small stocks and offset the market risk by going short on the large stocks. This can be achieved either through direct shorting of large stocks or through a short position in a futures contract such as the S&P 500 index that consists of large stocks. Of interest is that the equity hedge index shows negative factor loading on Fama and French's value (HML) factor, suggesting that the managers were long growth stocks during our sample period. This is not surprising, as growth stocks outperformed value stocks during this period. Finally, the equity hedge index also shows some exposure to commodities while the equity non-hedge index shows some exposure to MSCI emerging markets.

3.1.7 Significant risk exposures of HFR short selling index. Short selling strategy involves selling short overvalued securities with the hope of repurchasing them at lower prices in the future. Therefore one expects their factor loadings to be opposite in sign to those for managers using long positions, such as equity hedge and equity non-hedge. Our findings of negative betas on the market (Russell 3000 index), Fama and French's size (SMB) factors, and positive beta on Fama and French's value (HML) factor are in line with this expectation. Finally, the short selling index shows a payoff that resembles a short position in an OTM call option on the Russell 3000 index. This is again opposite to the short position in an OTM put option that we find in the other strategies, which are long the market. Negative beta on the Russell 3000 index along with this short position in the OTM call option suggests that short selling managers lose a lot during extremely bullish equity markets.

3.1.8 Summary of significant risk exposures of the HFR hedge fund indexes. Overall the evidence indicates that most hedge fund strategies exhibit a nonlinear risk-return relation as manifested through significant betas on option-based risk factors. In particular, the payoffs of event arbitrage, restructuring, event driven, relative value arbitrage, and convertible arbitrage strategies resemble that from writing a put option on the market index. This may be because these strategies relate to economic activity and lose money during large down moves in the equity market, or it may be because the managers, in order to improve their Sharpe ratio or to respond to their incentive contract, create (either directly or indirectly through dynamic trading) a payoff similar to that from writing a put option [see, e.g., Goetzmann et al. (2001), Lo (2001), and Siegmann and Lucas (2002)]. The risk exposures of event arbitrage and relative value arbitrage estimated using our approach are consistent with the findings of Mitchell and Pulvino (2001) and Gatev, Goetzmann, and Rouwenhorst (1999), who use detailed replication methodology to estimate the risk of these strategies.

3.2 Robustness checks

Before proceeding further, we examine the robustness of our results in terms of the choice of the database used and the choice of alternative strike prices for the construction of option-based factors.

3.2.1 Choice of database. Hedge Fund Research and CSFB/Tremont are two major hedge fund databases that have taken steps to account for the different biases, such as survivorship bias, in hedge funds [Fung and Hsieh (2000b, 2002b)]. One obvious question is how sensitive are the findings to the choice of database. To answer this question we repeat our analysis using the CSFB/Tremont indexes. The choice of index can potentially affect the results for reasons such as the extent of coverage, the method of index construction (e.g., equal weighting by HFR *vis-à-vis* value weighting by CSFB/Tremont), etc. We select four CSFB/Tremont strategies that are common with HFR, namely event driven, convertible arbitrage, long/short equity (equity hedge in the case of HFR), and dedicated short-bias (short selling in the case of HFR). We report the results from the regression in Equation (10) in Table 4.

Similar to HFR's event driven index, CSFB/Tremont's event driven index shows significant nonlinearity. In particular, its payoff resembles that from writing an OTM put option on the S&P 500 index. It also shows positive loading on Fama and French's size (SMB) and MSCI emerging market factors. For CSFB/Tremont's convertible arbitrage strategy, we find exposures to the lagged Russell 3000 index and the Lehman high yield index, suggesting the illiquid nature of the bonds and the credit risk involved in the strategy. For CSFB/Tremont's long/short equity strategy we find exposures that are very similar to those of HFR's equity hedge and equity non-hedge indexes. In particular, we find long exposure on the Russell 3000 index and Fama and French's size (SMB) and a short exposure to Fama and French's value (HML) factor. As expected, CSFB/Tremont's dedicated short-bias strategy shows negative loading on the Russell 3000 index and Fama and French's size (SMB) factor and a positive loading on Fama and French's value (HML) factor. These exposures are similar to those of HFR's short selling index. Overall both HFR and CSFB/Tremont indexes exhibit similar risk exposures that are consistent with the types of trading strategies the hedge funds claim to follow.

3.2.2 Choice of option strike prices. Since we find that a large number of hedge funds exhibit exposure similar to writing a put option on the market, it suggests that they bear significant tail risk. Hence we examine the robustness of our results by capturing even higher tail risk by specifying option-based strategies using deeper OTM options. In particular, we specify four different degrees of moneyness ranging from half a standard deviation to two standard deviations, where the standard deviation is computed using daily returns from the month immediately preceding the one for which the option returns are calculated. We observe that when one moves too far away from the ATM options, the contracts become illiquid and the prices become less reliable. We exercise caution by removing the outliers corresponding to the deeper OTM options and find results that are qualitatively similar.

The fact that the size factor turns out to be significant for a number of hedge fund strategies indicates that they invest in small stocks. It is possible that due to dynamic trading, the risk-return relationship with respect to small stocks may be nonlinear; in which case options on the S&P 500 composite index may not be able to capture this effect. Therefore we examine the robustness of our findings using options on Russell the 2000 index traded on the Chicago Mercantile Exchange. Unfortunately these contracts are highly illiquid and at times we are unable to find reasonable prices. However, for the period during which we observe reliable prices, we find results similar to those obtained with options on the S&P 500 composite index.

Finally, instead of using European-style options, we repeat our analysis with American-style three-month-to-maturity options on S&P 500 futures contracts and once again find qualitatively similar results. This suggests that our findings are robust to the inclusion of deeper OTM options, to the choice of a broader equity index and to the consideration of American-style options.

This concludes our discussion of the in-sample analysis of risk exposures of hedge funds. We now proceed to examine how well the in-sample risk exposures capture the out-of-sample performance of hedge funds.

4. Out-of-Sample Analysis of Hedge Fund Risk Exposures

If the risk exposures reported in Tables 4 and Table 5 are mere statistical artifacts of data, then these are unlikely to track hedge fund returns in an out-of-sample analysis. However, if they represent the true economic risks of different hedge fund strategies, then the replicating portfolios based on these factor loadings should do a good job of mimicking the out-of-sample performance of hedge funds. We examine this issue by constructing a replicating portfolio for each of the HFR and CSFB/Tremont indexes using the factor loadings obtained from our multifactor model. We compute the difference between the monthly return on the hedge fund index and that on the respective replicating portfolio. We conduct standard *t*-tests and Wilcoxon signed-rank tests to examine if the differences in the mean and median returns on the index and its respective replicating portfolio are statistically significant. We report the results in Table 6. We find the mean and median differences between the HFR and CSFB/Tremont indexes, and their replicating portfolios are statistically insignificant using both the *t*-test and the Wilcoxon signed-rank test, the only exception being CSFB/Tremont's convertible arbitrage index.

In general, the difference in the mean returns between the hedge fund indexes and the replicating portfolios from the model is about 24 basis points for the HFR indexes and about 94 basis points for the CSFB/Tremont indexes. Although this difference is not statistically significant in all except one case, it is nevertheless economically significant. A part of this difference can be attributed to survivorship and other biases [Fung and Hsieh (2000b, 2002b)]. The rest may be a compensation for bearing risks not captured by our model. Figure 1 graphically illustrates the returns on HFR indexes and those on the replicating portfolios during July 2000 to December 2001 period. It shows that the portfolios based on significant risk exposures estimated through our model closely track the

Table 5
Results with CSFB/Tremont value-weighted indexes

Event driven		Convertible arbitrage		Long/short equity		Short selling	
Factors	λ	Factors	λ	Factors	λ	Factors	λ
C	0.59	C	0.59	C	0.26	C	0.40
SPP_o	−0.66	LRUS	0.09	HML	−0.25	RUS	−1.03
SMB	0.08	SBW	−0.20	RUS	0.53	SMB	−0.42
MEM	0.08	LHY	0.41	SMB	0.31	DEFSPR	−0.32
LHY	0.50					MOM	0.22
SBG	−0.94					HML	0.19
DEFSPR	−0.46						
Adj-R^2	73.55	Adj-R^2	33.35	Adj-R^2	83.50	Adj-R^2	84.97

This table shows the results of the regression $R_t^i = c^i + \sum_{k=1}^{K} \lambda_k^i F_{k,t} + u_t^i$ for the four CSFB/Tremont indexes during the full sample period from January 1994 to June 2000. The table shows the intercept (C), statistically significant (at the 5% level) slope coefficients on the various buy-and-hold and option-based risk factors and adjusted R^2 (Adj-R^2). The buy-and-hold risk factors are the Russell 3000 index (RUS), lagged Russell 3000 index (LRUS), MSCI excluding the U.S. index (MXUS), MSCI emerging markets index (MEM), Fama and French size and book-to-market factors (SMB and HML), momentum factor (MOM), Salomon Brothers government and corporate bond index (SBG), Salomon Brothers world government bond index (SBW), Lehman high yield composite index (LHY), Federal Reserve Bank competitiveness-weighted dollar index (FRBI), Goldman Sachs commodity index (GSCI), and the change in the default spread in basis points (DEFSPR). The option-based risk factors include the at-the-money and out-of-the-money call and put options on the S&P 500 composite index ($SPC_{a/o}$ and $SPP_{a/o}$). For the two call and put option-based strategies, subscripts *a* and *o* refer to at-the-money and out-of-the-money, respectively.

Table 6
The *t-test* and Wilcoxon signed-rank test results for the difference in mean and median returns of HFR and CSFB/Tremont hedge fund indexes and their replicating portfolios during the out-of-sample period (July 2000 to December 2001)

		HFR		CSFB/Tremont	
Hedge fund strategy		*t*-test	Sign test	*t*-test	Sign test
Event arbitrage	Δr	−0.082	0.050		
	p-value	0.935	1.000		
Restructuring	Δr	−0.215	0.023		
	p-value	0.831	0.815		
Event driven	Δr	0.246	0.840	1.216	1.010
	p-value	0.808	1.000	0.238	0.096
Relative value arbitrage	Δr	−0.066	0.494		
	p-value	0.948	1.000		
Convertible arbitrage	Δr	1.988	0.516	2.265#	1.132#
	p-value	0.115	0.238	0.033	0.031
Equity hedge (long/short equity)	Δr	0.186	−0.161	0.450	0.377
	p-value	0.854	0.481	0.657	0.481
Equity non-hedge	Δr	−0.220	−0.516		
	p-value	0.827	0.815		
Short selling (dedicated short-bias)	Δr	0.035	−0.469	−0.168	−1.918
	p-value	0.973	0.815	0.868	0.815

This table shows the results of two-sided heteroscedastic *t*-test and Wilcoxon signed-rank test for the difference in the mean and median returns of eight HFR and four CSFB/Tremont indexes and those of their corresponding replicating portfolios using our model (i.e., using both buy-and-hold and option-based risk factors) during the out-of-sample period from July 2000 to December 2001. Δr is the mean (median) return of the index minus that of its replicating portfolio for the *t*-test and Wilcoxon signed-rank test, respectively. # indicates Δr is significantly different from zero at the 5% level.

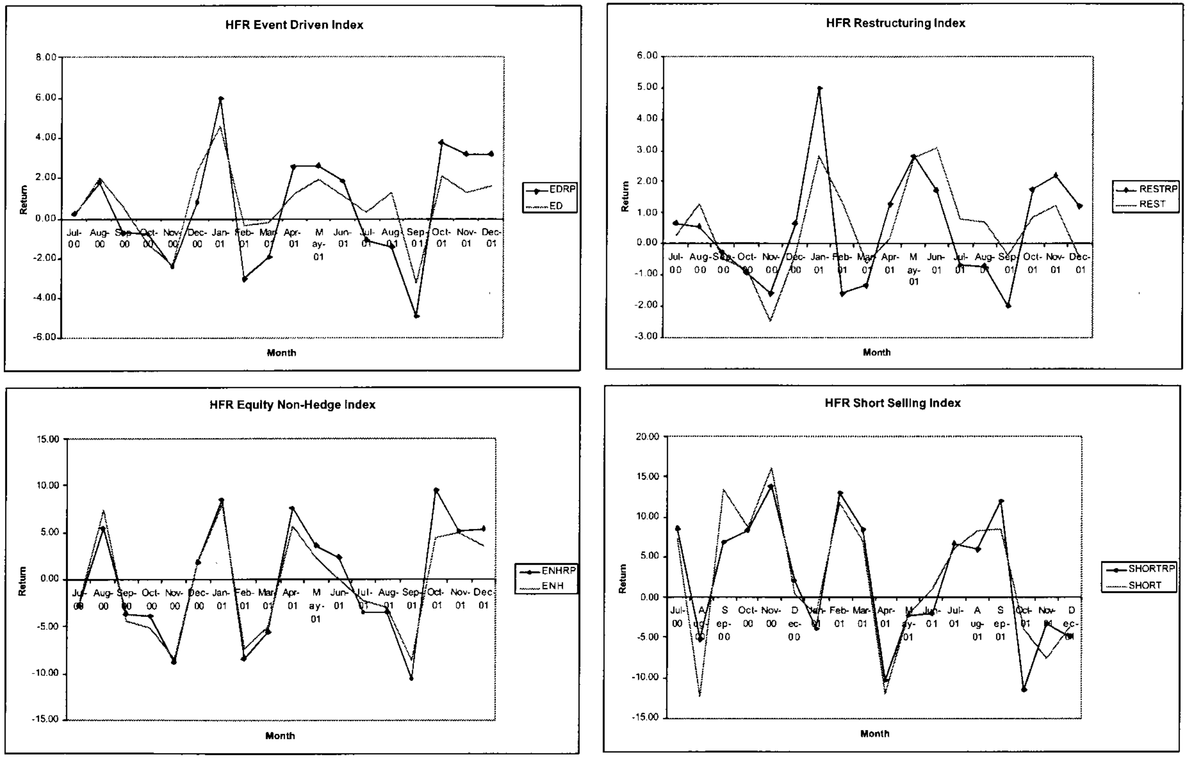


Figure 1
Out-of-sample results for HFR strategies
This figure plots the returns for the replicating portfolios and the actual HFR index returns during the out-of-sample period from July 2000 to December 2001. EDRP, RESTRP, HLBRP, and SHORTRP are the replicating portfolios for HFR's event driven (ED), restructuring (REST), equity non-hedge (ENH), and short selling (SHORT) hedge fund strategies constructed using buy-and-hold and option-based risk factors estimated during our sample period from January 1990 to June 2000.

hedge fund returns during the out-of-sample period. This suggests that our approach is able to capture the dominant economic risk exposures of hedge funds. Since investors invest in individual hedge funds, we repeat the out-of-sample analysis with individual hedge fund returns and report the findings in Appendix A.

A wide range of hedge fund strategies exhibiting nonlinear payoffs has important implications for portfolio decisions involving hedge funds. We investigate this issue in the next section.

5. Portfolio Decisions with Hedge Funds

Our results from Section 3 show that the payoffs on a wide range of hedge fund indexes resemble those from selling OTM put options on the market index. This suggests that these hedge funds may be selling portfolio insurance, a strategy providing positive returns when the market does not lose much and experiencing large losses in extreme down-market conditions. Hedge funds market themselves as absolute return vehicles, which aim to deliver positive returns irrespective of the market conditions. Arguably hedge fund investors care about the absolute value of losses (and not losses relative to a benchmark index). Therefore a portfolio construction framework involving hedge funds must explicitly account for large losses (i.e., the tail risk of hedge funds) in down-market conditions. Fung and Hsieh (1999) argue that asset allocation involving hedge funds should not be based on the mean-variance framework, as it is appropriate only for normally distributed returns or for quadratic preferences of the investors. They show that although the rankings based on the mean-variance criterion are approximately correct, risk assessment and management based on such a criterion will not be correct since it does not take into account the probability of large negative returns. Our results from Section 3 show that hedge fund payoffs are nonlinear and asymmetric with significant negative tail risk. Therefore any portfolio constructed involving hedge funds needs to explicitly account for their tail risk, an important issue that we address in this section.

The Basle Committee on Banking Supervision has recommended use of a risk management framework such as VaR to better understand and manage the downside risk. Hull (2000, p. 342) reports a number of financial institutions, corporate treasurers, and fund managers use VaR. However, researchers such as Artzner et al. (1999) have shown that VaR has problematic properties (nonsubadditive, nonconvex, nondifferentiable, etc.) and have proposed the use of CVaR, which equals the statistical mean of the losses exceeding the VaR and which is closely related to Basak and Shapiro's (2001) limited expected loss measure. While the VaR focuses only on the frequency of extreme events, CVaR focuses on both the frequency and size of losses in case of extreme events.

5.1 Theoretical framework for VaR and CVaR

In this section we define the concepts of VaR and CVaR by evaluating the risk beyond the VaR using simple statistics. Let the return on a portfolio over a given period of time is denoted by R. Let the probability density function (PDF) of R be denoted by f_R and the cumulative distribution function (CDF) be denoted by F_R. We denote the VaR of the portfolio for a probability level p as VaR (F_R, p) in order to indicate its dependence on the CDF and the specified probability level. When expressed as a percentage of the initial value of the portfolio and as a positive number, the VaR of the portfolio can be expressed as

$$\mathrm{VaR}(F_R,p) = -F_R^{-1}(1-p). \tag{6}$$

The CVaR measures the expectation of the losses greater than or equal to the VaR and is given by the ratio of the size of the losses beyond the VaR to the frequency of losses greater than or equal to the VaR. It can be expressed as

$$\mathrm{CVaR}(F_R,p) = -E(R|R \leq -\mathrm{VaR}) = -\frac{\int_{-\infty}^{-\mathrm{VaR}} z f_R(z)dz}{F_R(-\mathrm{VaR})}. \tag{7}$$

Considering the various advantages of CVaR over VaR, we use CVaR as a risk management tool to control the tail risk of a portfolio involving hedge funds. While optimizing, one can either impose a distributional assumption on the security returns or use the empirical distribution of security returns. Since CVaR focuses on the tail risk, considering parameterized distributions may not be able to fully capture this risk due to their potentially poor tail properties. Therefore we use the empirical distribution of hedge fund returns for M-CVaR optimization.[14]

5.2 Mean-variance and M-CVaR optimization results

As the mean-variance framework implicitly assumes normality of asset returns, it is likely to underestimate the tail risk for assets with negatively skewed payoffs. In this section we test this conjecture by using the M-CVaR framework theorized above. Specifically, we compare the tail losses on mean-variance optimal portfolios with those on the M-CVaR optimal portfolios for different confidence levels. In particular, we construct a mean-variance efficient frontier and a M-CVaR efficient frontier using the eight HFR hedge fund strategies. We compute the CVaRs of the mean-variance efficient portfolios of different volatilities and compare

[14] We follow Palmquist, Uryasev, and Krokhmal (1999) and Alexander and Baptista (2002) to construct the M-CVaR frontier. It turns out to be a linear programming problem which we solve using MATLAB's *linprog* function. For more details of formulating the M-CVaR optimization problem as a linear programming problem see Rockafellar and Uryasev (2000) and links provided at www.ise.ufl.edu/uryasev.

them with those of M-CVaR efficient portfolios with volatilities. We also measure the differences in their mean returns, which indicate how much of the return one has to give up for reducing the tailrisk.

Table 7 reports the CVaRs of mean-variance and M-CVaR efficient portfolios at 90%, 95%, and 99% confidence levels. It also reports ratios of the CVaRs and differences in mean returns of the two portfolios. As expected, CVaR increases with the portfolio volatility and confidence level (due to going out further in the left tail at a higher confidence level). The average ratio of CVaR of mean-variance and M-CVaR portfolios ranges from 1.12 at a 90% confidence level to 1.54 at a 99% confidence level. This suggests that tail risk is significantly underestimated using the mean-variance approach, the range of underestimation being 12% to 54% for a confidence level ranging from 90% to 99%. This is an economically significant number considering that if a hedge fund is managing $1 billion, if the CVaR of a M-CVaR efficient portfolio is 1% at a 99% confidence level, the average loss can exceed $10 million in 1 of 100 cases, while using a mean-variance approach the average loss can exceed $15.4 million at the same confidence level.

Figure 2 illustrates how the ratio of CVaR of a mean-variance efficient portfolio to the CVaR of a M-CVaR efficient portfolio of hedge funds varies with the portfolio volatility. As mentioned earlier, it is clear from the figure that the ratio is higher for higher confidence levels. However, the ratio decreases with increasing portfolio volatility, suggesting that for efficient portfolios of high volatility, the underestimation of loss due to use of the mean-variance approach is less.[15] In general, the mean-variance approach underestimates the loss compared to the M-CVaR approach and this underestimation is substantial for portfolios with low volatility. The differences in mean returns reported in Table 7, which can be thought of as the price investors pay to reduce tail risk, are consistent with this, they are higher for portfolios with low volatility. For 90% and 95% confidence levels, the difference in mean returns is up to 7 basis points, while at a 99% confidence level it is up to 17 basis points.[16]

Having compared and contrasted the differences between efficient portfolios constructed using mean-variance and M-CVaR approach, we now proceed with the examination of long-run risk-return trade-offs of hedge funds.

[15] This result seems to be consistent with Alexander and Baptista (2002) who find that the mean-variance efficient portfolios with smaller standard deviations may not be efficient in the mean-conditional expected loss (CEL) space. As mentioned earlier, their CEL measure is equivalent to our CVaR measure.

[16] There are two ways in which investors can buy insurance to reduce the left-tail risk. One involves buying deep OTM put options on the equity market, while the other involves including trend-following strategies in a portfolio of hedge funds. In case of a downturn in equity markets, the put option will deliver positive returns. However, the writer of the put option will have to short the equities in order to dynamically hedge the exposure, which can further drive down the equity prices. This is not the case with trend followers who deliver positive returns when equity markets are down, but do so by trading in markets other than equity, like currencies and interest rate markets [see Fung and Hsieh (2001)].

Table 7
Conditional Value-at-Risk for mean-variance and mean-conditional Value-at-Risk efficient portfolios

	90%				95%				99%			
s	CVaR (M-V)	CVaR (M-CVaR)	Ratio	Δm	CVaR (M-V)	CVaR (M-CVaR)	Ratio	Δm	CVaR (M-V)	CVaR (M-CVaR)	Ratio	Δm
0.73	0.42	0.33	1.25	−7.04	0.88	0.51	1.75	−7.24	2.41	0.88	2.73	−17.15
0.74	0.35	0.29	1.23	−5.51	0.85	0.51	1.67	−5.73	2.52	0.97	2.60	−14.73
0.76	0.33	0.27	1.22	−4.10	0.82	0.53	1.55	−4.30	2.63	1.07	2.45	−12.47
0.80	0.33	0.27	1.21	−2.65	0.81	0.55	1.48	−2.90	2.74	1.29	2.12	−10.85
0.85	0.33	0.28	1.17	−1.30	0.84	0.60	1.40	−1.55	2.84	1.49	1.91	−9.26
0.92	0.36	0.31	1.15	−1.16	0.90	0.69	1.30	−1.36	2.88	1.70	1.70	−7.72
1.00	0.43	0.38	1.14	−1.04	0.97	0.77	1.25	−1.14	2.86	1.90	1.50	−6.40
1.10	0.51	0.45	1.12	−0.93	1.05	0.88	1.20	−0.96	2.83	2.10	1.35	−5.10
1.21	0.61	0.55	1.11	−0.81	1.20	1.04	1.15	−0.85	2.83	2.36	1.20	−4.90
1.33	0.75	0.68	1.10	−0.68	1.39	1.25	1.11	−0.72	2.86	2.51	1.14	−2.30
1.47	0.91	0.84	1.08	−0.54	1.59	1.48	1.08	−0.58	3.18	2.94	1.08	−1.34
1.61	1.11	1.05	1.06	−0.40	1.87	1.76	1.06	−0.44	3.54	3.31	1.07	−1.01
1.78	1.38	1.31	1.05	−0.26	2.16	2.04	1.06	−0.30	4.31	4.07	1.06	−0.74
1.97	1.68	1.59	1.05	−0.14	2.49	2.35	1.06	−0.21	5.09	4.85	1.05	−0.51
2.17	1.98	1.89	1.05	−0.08	2.88	2.73	1.06	−0.12	5.86	5.58	1.05	−0.23
2.37	2.30	2.20	1.04	−0.03	3.28	3.12	1.05	−0.08	6.64	6.38	1.04	−0.12
2.59	2.61	2.51	1.04	−0.01	3.67	3.51	1.05	−0.04	7.41	7.13	1.04	−0.07
Avg.			1.12				1.25				1.54	

This table shows the conditional Value-at-Risk (CVaR) figures (reported as the magnitude of losses) at different confidence levels for mean-variance (M-V) and mean-CVaR (M-CVaR) efficient portfolios constructed using monthly returns of the eight HFR hedge fund strategies from January 1990 to June 2000. s indicates the volatility of portfolio returns and Ratio is the ratio of CVaR of M-V efficient portfolio to that of M-CVaR efficient portfolio for the same portfolio volatility. Δm is the difference in the mean returns (in basis points) of the M-CVaR and M-V efficient portfolios for the same portfolio volatility, s.

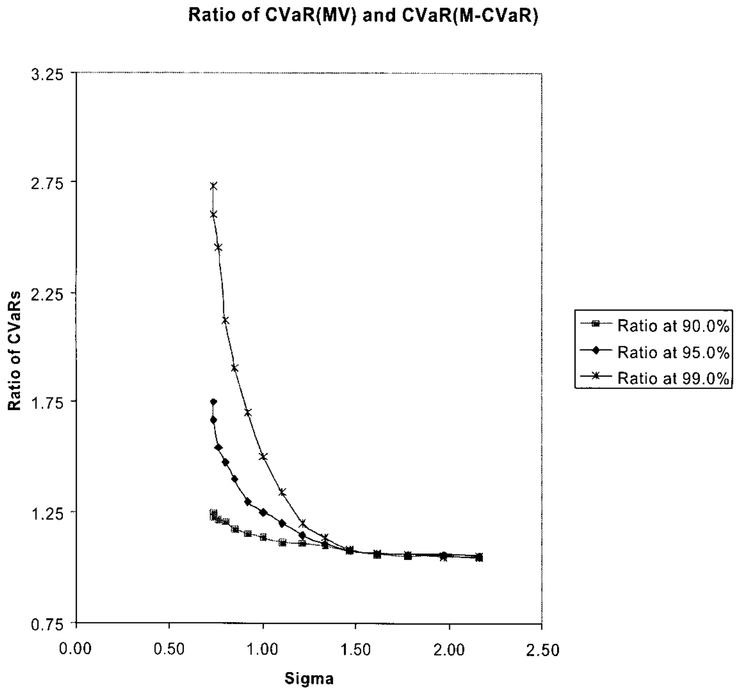


Figure 2
Ratio of conditional Value-at-Risk for mean-variance and mean-conditional Value-at-Risk efficient portfolios
This figure plots the ratio of the conditional Value-at-Risk (CVaR) for mean-variance and mean-CVaR efficient portfolios at different confidence levels for different levels of portfolio volatility. The efficient portfolios are constructed using monthly returns of eight HFR hedge fund strategies during our sample period from January 1990 to June 2000.

6. Long-Run Performance of Hedge Funds

One of the limitations investors face when dealing with hedge funds is that the return history of hedge fund indexes goes back at most to January 1990. One way to circumvent this limitation is to work with the underlying risk factors for which longer return histories are available. For example, data on market, size, value, and momentum factors is available from 1927. For the option-based factors, although returns data are available only from 1982, it is possible to construct a theoretical return series going back to 1927 using Black and Scholes' (1973) formula.[17] This provides us with

[17] We use historical volatility (based on a five-year rolling window) to compute the option prices. For the first five years we use average volatility during the five-year period. We compute returns based on theoretical prices for the 1927 to 1982 period and based on market prices for the remaining period.

the return history of key risk factors going back to 1927. In order to shed light on the long-run performance, we regress the hedge fund index returns on market, size, value, momentum, and option-based risk factors and reestimate the factor loadings. Using these factor loadings, we recompute the returns of the hedge fund index, replicating portfolios from January 1927 to December 1989. We call these the long-run systematic returns of different hedge fund strategies. In order to compare returns on a like basis, we also recompute systematic returns to the indexes during the recent period (January 1990 to June 2000) using the simplified model. We report the summary statistics of these returns for the HFR indexes in Table 8.

We find interesting differences between the recent returns and long-run systematic returns. For the HFR indexes, the mean long-run (recent) monthly return varies from 0.0% (0.15%) for the short selling strategy to 0.97% (1.26%) for the restructuring strategy. The corresponding volatility ranges from 1.45% (0.88%) for the event arbitrage strategy to 6.27% (5.81%) for the short selling strategy. The magnitude of long-run CVaRs at the 90%, 95%, and 99% levels across the eight HFR indexes are higher on average by 100%, 60%, and 40%, respectively, than the corresponding recent period CVaRs. The findings with the CSFB/Tremont indexes are similar as well (see Table 9). For the CSFB/Tremont indexes, the mean long-run (recent) monthly return varies from −0.18% (−0.55%) for the short selling strategy to 0.83% (1.26%) for the event driven strategy. The corresponding volatility ranges from 1.00% (0.68%) for the convertible arbitrage strategy to 6.65% (4.88%) for the short selling strategy. The magnitude of long-run CVaRs at the 90%, 95%, and 99% levels are higher on average by 90%, 70%, and 100% respectively, than the corresponding recent period CVaRs. Overall, across all the indexes, we find that the long-run returns are smaller, the long-run volatilities are larger, and the magnitude of long-run CVaRs are larger compared to the recent period.

In order to examine whether the differences in the long-run returns and volatilities are statistically significant from those in the recent period, we conduct the standard *t*-test (for the means), Wilcoxon signed-rank test (for the median), and variance ratio test (for standard deviations). We report the findings in Table 10. For all the HFR indexes, we find that the mean long-run returns are smaller than those for the recent period by about 23 basis points per month (or 2.76% per annum) and this difference is statistically significant in three cases.[18] The long-run median returns are also smaller than those during the recent period by about 25 basis points (or 3.00 percent per annum) and the difference is statistically significant for three indexes. The long-run volatilities are also significantly larger

[18] Even in cases where the difference is not statistically significant, a figure ranging from 2.50% to 3.00% per annum is economically significant.

Table 8
Summary statistics of systematic returns of HFR hedge fund indexes

Hedge fund strategy	Mean	SD	Median	Minimum	Maximum	CVaR (90%)	CVaR (95%)	CVaR (99%)
Panel A: Recent returns—January 1990 to June 2000								
Nondirectional								
Event arbitrage	1.00	0.88	1.18	−3.31	2.40	1.00	1.86	3.31
Restructuring	1.26	1.49	1.53	−5.30	4.88	1.83	3.10	5.30
Event driven	1.08	1.61	1.50	−6.66	4.40	2.25	3.54	6.66
Relative value arbitrage	0.82	0.89	0.94	−3.22	3.03	0.91	1.62	3.22
Convertible arbitrage	0.83	0.65	0.91	−1.90	1.99	0.46	0.95	1.90
Equity hedge	0.81	2.24	0.89	−8.54	7.82	3.16	4.46	8.54
Directional								
Equity non-hedge	1.17	3.90	1.61	−16.11	10.08	6.22	8.37	16.11
Short selling	0.15	5.81	0.10	−18.54	20.95	9.95	12.78	18.54
Panel B: Long-run returns—January 1927 to December 1989								
Nondirectional								
Event arbitrage	0.72	1.45	0.95	−7.76	7.81	2.45	3.47	5.71
Restructuring	0.97	2.40	1.25	−11.11	18.78	3.99	5.56	8.53
Event driven	0.85	2.64	1.16	−11.73	19.94	4.38	5.96	9.18
Relative value arbitrage	0.61	1.46	0.70	−6.37	10.16	2.23	3.12	5.12
Convertible arbitrage	0.57	0.97	0.66	−3.97	6.57	1.41	1.97	3.05
Equity hedge	0.60	2.69	0.66	−11.70	19.32	4.26	5.71	9.30
Directional								
Equity non-hedge	0.96	5.53	1.20	−23.43	39.87	8.95	11.77	18.82
Short selling	0.00	6.27	0.05	−39.72	26.94	11.08	14.76	25.94

This table shows the mean returns, standard deviations (SD), medians, minimum realizations, maximum realizations, and Conditional Value-at-Risk (reported as the magnitude of losses) at 90%, 95%, and 99% confidence levels for the systematic returns of eight HFR hedge fund indexes during the sample period from January 1990 to June 2000 (panel A) and before our sample period from January 1927 to December 1989 (panel B).

than those in the recent period in seven out of eight cases. The results for the CSFB/Tremont indexes are qualitatively similar. For all strategies except short selling, the long-run mean and median returns are smaller than those during the recent period, and the difference is statistically significant in the case of two indexes for mean returns and one index for median returns. The long-run volatilities are also significantly larger than those in the recent period in three out of the four cases.[19] Overall, these findings suggest that the performance of hedge funds during the recent period appears significantly better compared with their long-run performance.

[19] In order to make the HFR results comparable with those from CSFB/Tremont, we divide the HFR sample period (January 1990 to June 2000) into two subperiods, January 1990 to December 1993 and January 1994 to June 2000, for the second subperiod to coincide with that of CSFB/Tremont. We find that the difference in the mean and median returns over the long run and those during the second subperiod are 20 and 21 basis points, figures comparable with the 23 and 25 basis points we find using January 1990 to June 2000 period. Also, the magnitude of CVaRs during the second subperiod compared with that during the long run are 100%, 70%, and 40% lower, figures comparable with the 100%, 60%, and 40%, respectively, we find using the January 1990 to June 2000 period.

Table 9
Summary statistics of systematic returns of CSFB/Tremont hedge fund indexes

Hedge fund strategy	Mean	SD	Median	Minimum	Maximum	CVaR (90%)	CVaR (95%)	CVaR (99%)
Panel A: Recent returns—January 1994 to June 2000								
Nondirectional								
Event driven	1.26	1.56	1.56	−6.29	4.16	1.85	2.98	6.29
Convertible arbitrage	0.91	0.68	1.04	−1.57	1.84	0.49	0.97	1.57
Long/short equity	1.16	3.38	1.01	−11.61	10.86	5.05	7.00	11.61
Directional								
Dedicated short-bias	−0.55	4.88	−0.83	−9.73	21.60	7.28	8.26	9.73
Panel B: Long-run returns—January 1927 to December 1993								
Nondirectional								
Event driven	0.83	2.27	1.17	−10.83	15.60	3.92	5.44	8.65
Convertible arbitrage	0.59	1.00	0.70	−4.31	5.50	1.45	2.13	3.36
Long/short equity	0.62	3.23	0.77	−15.35	18.52	5.38	7.12	12.02
Directional								
Dedicated short-bias	−0.18	6.65	−0.41	−55.01	29.85	11.64	16.08	33.86

This table shows the mean returns, standard deviations (SD), medians, minimum realizations, maximum realizations, and conditional Value-at-Risk (reported as the magnitude of losses) at 90%, 95%, and 99% confidence levels for the systematic returns of four CSFB/Tremont hedge fund indexes during the sample period (January 1994 to June 2000) (panel A) and before the sample period from January 1927 to December 1993 (panel B).

Table 10
The *t*-test, Wilcoxon signed-rank test, and variance ratio test results for the difference in mean, median, and standard deviation of systematic returns of HFR and CSFB/Tremont hedge fund indexes

		HFR			CSFB/Tremont		
Hedge fund strategy		*t*-test	Sign test	VR test	*t*-test	Sign test	VR test
Event arbitrage	Δ	−0.278*	−0.231*	0.559*			
	p-value	0.004	0.000	0.000			
Restructuring	Δ	−0.288	−0.274	0.904*			
	p-value	0.071	0.247	0.000			
Event driven	Δ	−0.229	−0.342	1.029*	−0.433*	−0.397	0.714*
	p-value	0.186	0.247	0.000	0.028	0.428	0.000
Relative value arbitrage	Δ	−0.211*	−0.243*	0.578*			
	p-value	0.026	0.002	0.000			
Convertible arbitrage	Δ	−0.252*	−0.242*	0.322*	−0.318*	−0.339*	0.321*
	p-value	0.000	0.001	0.000	0.000	0.004	0.000
Equity hedge (long/short equity)	Δ	−0.210	−0.237	0.448*	−0.538	−0.235	−0.152
	p-value	0.347	0.789	0.008	0.162	0.428	0.588
Equity non-hedge	Δ	−0.207	−0.409	1.636*			
	p-value	0.607	0.789	0.000			
Short selling (dedicated short-bias)	Δ	−0.146	−0.043	0.457	0.374	0.421	1.779*
	p-value	0.808	0.247	0.269	0.534	0.428	0.000

This table shows the results of two-sided heteroscedastic *t*-test, Wilcoxon signed-rank test, and variance ratio (VR) test for the difference in the mean, median, and standard deviation of systematic returns of eight HFR and four CSFB/Tremont indexes during the presample period (January 1927 to December 1989 for HFR and January 1927 to December 1993 for CSFB/Tremont) and those during the sample period (January 1990 to June 2000 for HFR and January 1994 to June 2000 for CSFB/Tremont). Δ is the difference in the mean (*t*-test), median (sign test) and standard deviation (VR test) of the systematic returns during the presample and sample period.

*Indicates that the difference Δ is significantly different from zero at 10% level.

7. Conclusion

In this article we characterize the linear and nonlinear risks of a wide range of hedge fund strategies using buy-and-hold and option-based risk factors. For this purpose we employ a two-step approach. In the first step we estimate the factor loadings of hedge funds using the returns on standard asset classes and options on them as factors. We construct replicating portfolios that best explain the in-sample variation in hedge fund index returns. In the second step we examine how well these replicating portfolios capture the out-of-sample performance of hedge funds. We conduct the analysis both at the index level as well as at an individual level.

We have four main results. First, we find that it is important to allow for a nonlinear risk-return relation while analyzing hedge funds. Along with the nonlinear exposure to the equity market index, we find that hedge funds also exhibit significant risk exposures to Fama and French's (1993) size and value factors and Carhart's (1997) momentum factor. Second, we observe that a wide range of hedge fund strategies exhibit returns similar to those from writing a put option on the equity index. The observed nonlinearities across multiple strategies suggest that these events are not statistical outliers, but represent important risks borne by hedge fund investors. Third, since hedge funds exhibit significant left-tail risk, we compare and contrast the tail losses of portfolios constructed using the mean-variance framework and mean-conditional value-at-risk framework. We find that using the traditional mean-variance framework substantially underestimates the tail losses and this underestimation is most severe for portfolios with low volatility. Finally, we compare and contrast the long-run systematic returns of hedge funds with those observed during the recent period. Across almost all hedge fund indexes we find that the long-run returns are lower, the long-run volatilities are higher, and the long-run tail losses are larger compared with those during the recent period.

Understanding the risk exposures of hedge funds is an important area of research. We need a better understanding of this issue while making investment management decisions involving hedge funds. Unfortunately this is a tricky issue, since hedge funds provide limited disclosure. In this context, our approach provides useful information to investors dealing with portfolio construction and risk management-related issues. At a more general level, it indicates whether a fund has been classified correctly or not and, when applied on an ongoing basis, it enables investors to address issues like hedge fund style drift. Estimation of hedge fund risks is also important, as a large number of hedge funds propose a risk-free rate as a benchmark for claiming incentive fees. This would be appropriate only if they carried no systematic risks. However, we find that a large majority of hedge funds carry a significant amount of systematic risk. We

Table 11
Out-of-sample regression results with individual hedge funds using replicating portfolios

	HFR Number of funds								TASS+ Number of funds			
Range of R^2	EA	REST	ED	RVAL	CA	EH	ENH	SS	ED	CA	L-S E	DSB
Less than −20%	0	0	1	0	0	0	0	0	0	0	1	0
−20–−10%	5	0	3	0	1	8	1	0	7	5	30	0
−10–0%	7	6	48	7	29	99	9	0	12	5	34	1
0–10%	5	3	14	0	10	28	3	0	11	2	30	0
10–20%	1	4	11	2	8	29	4	1	9	6	23	0
20–30%	3	3	8	0	5	39	4	1	5	2	17	0
30–40%	0	1	7	1	0	37	7	1	9	0	21	0
40–50%	0	1	13	1	2	35	2	0	6	3	27	0
50–60%	0	0	12	0	2	40	7	2	6	2	17	0
60–70%	0	0	2	1	1	44	7	4	4	3	13	1
70–80%	0	1	2	0	0	35	7	4	3	3	20	1
80–90%	0	0	1	0	0	21	5	2	2	1	14	1
90–100%	0	0	0	0	0	1	2	0	0	0	2	2
Mean	0.3	13.9	15.4	9.6	6.0	32.1	41.7	60.9	23.8	24.8	27.5	67.9
Median	−5.0	11.0	5.6	−5.8	−2.8	31.8	43.1	61.0	19.2	18.1	24.1	81.1

This table shows the distribution of the adjusted-R^2 (in terms of the number of funds falling in different ranges of R^2 values, mean and median R^2 values) from the following out-of-the-sample regressions:

$$R^i_{j,t} = \alpha^i + \beta^i RP_{j,t} + e^i_t,$$

where $R^i_{j,t}$ is the net-of-fees excess return (in excess of the risk-free rate of interest) on an individual hedge fund i belonging to hedge fund strategy j during month t, and $RP_{j,t}$ is the excess return on the replicating portfolio to strategy j during month t. We consider individual hedge funds following eight different strategies [event arbitrage (EA), restructuring (REST), event driven (ED), relative value arbitrage (RVAL), convertible arbitrage (CA), equity hedge (EH), equity non-hedge (ENH) and short selling (SS)] from the HFR database on the excess returns of the HFR hedge fund index replicating portfolios during the July 2000 to August 2001 period and individual hedge funds following four different strategies [event driven (ED), convertible arbitrage (CA), long/short equity (L-SE), and dedicated short-bias (DSB)] from TASS+ database on the CSFB/Tremont hedge fund index replicating portfolios during the July 2000 to August 2001 period.

believe our findings raise important concerns relating to issues like benchmark design and manager compensation.[20] In addition, our analysis provides a tool for measuring the net and gross risk exposures of hedge funds. This can help address regulators' concerns regarding the potential risk hedge funds can pose to stability of financial markets.

The popular press classifies some hedge fund strategies as short-volatility strategies. The short positions in put options that we find are consistent with this notion. If one can locate or construct an instrument whose payoff is directly related to volatility of financial markets, then it would be interesting to include it as an additional asset-class factor. Similarly it would also be interesting to create proxies that capture returns from

[20] Previous researchers including Brown, Goetzmann, and Ibbotson (1999) and Agarwal and Naik (2000), examining persistence in hedge fund managers' performance, have used a peer group average as a benchmark to adjust for systematic risk. It would be interesting to examine persistence in performance after adjusting for systematic risk using our model.

Table 12
Out-of-sample regression results with individual hedge funds using indexes

	HFR Number of funds								TASS+ Number of funds			
Range of R^2	EA	REST	ED	RVAL	CA	EH	ENH	SS	ED	CA	L-SE	DSB
Less than −20%	0	0	0	0	0	0	0	0	0	0	1	0
−20–−10%	0	0	4	0	1	9	1	0	10	3	43	0
−10–0%	2	5	39	4	7	77	10	0	11	5	48	0
0–10%	3	2	18	2	6	52	1	0	12	3	31	1
10–20%	6	3	11	2	10	30	5	1	4	3	20	0
20–30%	1	3	12	1	4	36	4	0	6	2	17	0
30–40%	3	4	7	1	5	23	4	1	9	5	14	1
40–50%	1	0	15	1	3	47	7	0	6	1	14	0
50–60%	1	1	6	0	6	45	5	3	8	1	14	0
60–70%	1	1	8	0	9	43	5	3	6	3	19	0
70–80%	0	0	1	1	4	41	6	1	0	5	17	2
80–90%	2	0	1	0	3	12	9	3	1	1	10	1
90–100%	1	0	0	0	0	1	1	3	1	0	1	1
Mean	31.0	18.9	17.9	16.1	35.0	32.2	41.8	68.6	22.1	31.1	21.4	59.8
Median	18.9	17.8	9.5	8.4	30.9	32.3	45.1	66.4	17.7	28.9	11.2	75.4

This table shows the distribution of the adjusted-R^2 (in terms of the number of funds falling in different ranges of R^2 values, mean and median R^2 values) from the following out-of-the-sample regressions:

$$R^i_{j,t} = \alpha^i + \beta^i I_{j,t} + e^i_t,$$

where $R^i_{j,t}$ is the net-of-fees excess return (in excess of the risk-free rate of interest) on an individual hedge fund i belonging to hedge fund strategy j during month t, and $I_{j,t}$ is the excess return on the index for strategy j during month t. We consider individual hedge funds following eight different strategies [Event driven (ED), relative value arbitrage (RVA), equity hedge (EH), equity non-hedge (ENH), short selling (SS), event arbitrage (EA), and restructuring (REST)] from HFR database on the excess returns of the HFR hedge fund index replicating portfolios during the July 2000 to August 2001 period and individual hedge funds following four different strategies [Event driven (ED), convertible arbitrage (CA), long/short equity (L-SE), and dedicated short-bias (DSB)] from TASS+ database on the CSFB/Tremont hedge fund index replicating portfolios during the July 2000 to August 2001 period.

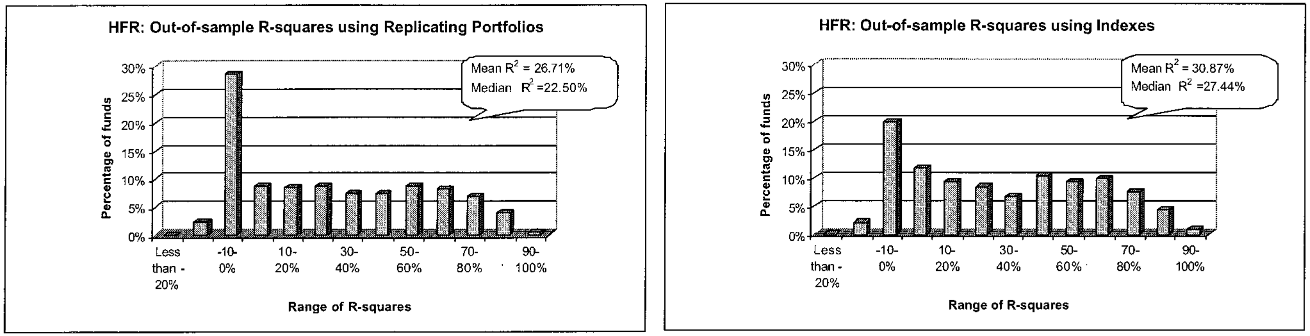
arbitrage opportunities. For example, one could use a statistical arbitrage model and compute returns to arbitraging mispriced securities. Returns to such strategies can also be used as additional factors in our model to capture some of the active (i.e., nonsystematic) risk of hedge funds. These issues are a part of our ongoing research agenda.

Appendix A: Out-of-Sample Analysis with Individual Hedge Funds

Our analysis in Section 4 is at the hedge fund index level. Since investors invest in individual hedge funds, we also examine how well our replicating portfolios are able to explain the out-of-sample variation in individual hedge funds compared to the hedge fund indexes themselves. Toward that end, we regress the returns of individual hedge funds belonging to the different indexes on our replicating portfolios for those indexes during the July 2000 to August 2001 period.[21] We report in Table 11 the distribution of adjusted R^2 obtained with

[21] We only consider those individual hedge funds that have at least six monthly returns during the July 2000 to June 2001 period. For the CSFB/Tremont database, individual funds following the "long/short equity" strategy are classified under the "long/short equity hedge" category.

Panel A: Results for individual HFR hedge funds



Panel B: Results for individual CSFB/Tremont hedge funds

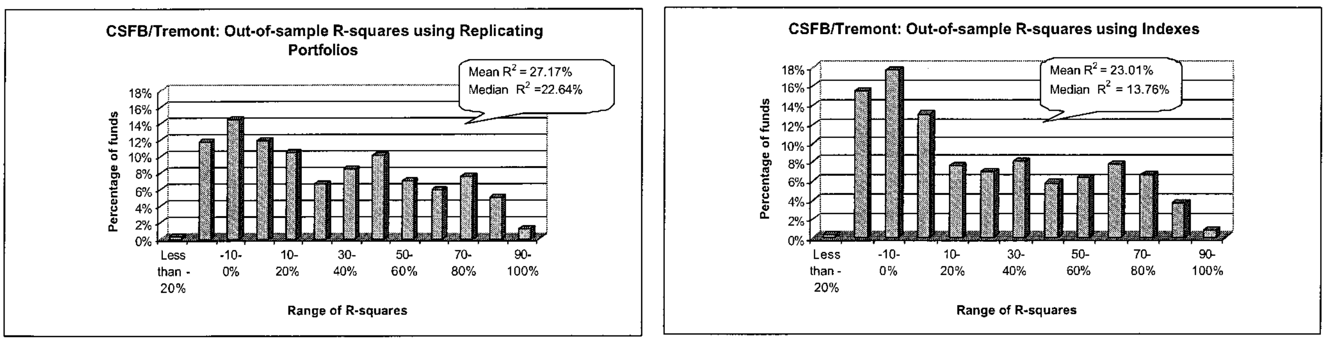


Figure 3.
Distribution of out-of-sample R^2 for individual HFR and CSFB/Tremont hedge funds
These figures show the distribution of out-of-sample R^2 from regressions of the excess returns on individual hedge funds in HFR and CSFB/Tremont databases on the excess returns of their corresponding index replicating portfolios and on the excess returns of their corresponding indexes.

our HFR and CSFB/Tremont index-replicating portfolios. In order to compare how well our index-replicating portfolios are able to explain the out-of-sample variation in individual hedge fund returns, we need to know how well the hedge fund indexes to which they belong explain their returns in the first place. For this purpose we also regress the returns of individual hedge funds on the respective HFR and CSFB/Tremont indexes. We report in Table 12 the distribution of adjusted R^2 of these regressions. In Figure 3 we plot the histogram of adjusted R^2 from the regressions using HFR and CSFB/Tremont replicating portfolios and indexes.

As can be seen from Table 11, our replicating portfolios exhibit mean (median) adjusted R^2 ranging from 0.3% to 60.9% (−5.0% to 61.0%) for HFR and 23.8% to 67.9% (18.1% to 81.1%) for CSFB/Tremont funds. This range of mean and median adjusted R^2 is similar to those obtained using the respective HFR and CSFB/Tremont hedge fund indexes. As shown in Table 12, indexes exhibit mean (median) adjusted R^2 ranging from 16.1% to 68.6% (8.4% to 66.4%) for HFR and 21.4% to 59.8% (11.2% to 75.4%) for CSFB/Tremont funds. Overall, the replicating portfolios explain an average of 26.7% (median 22.5%) of variation in out-of-sample returns of individual HFR funds and an average of 27.2% (median 22.6%) of variation in the out-of-sample returns of individual CSFB/Tremont funds. The corresponding figures for the indexes are mean (median) adjusted R^2 of 30.9% (27.4%) for HFR and 23.0% (13.8%) for CSFB/Tremont. These figures are very much comparable to those we obtain using replicating portfolios. In fact, for CSFB/Tremont funds overall, our replicating portfolios do a slightly better job than the indexes in explaining the variation in out-of-sample returns of individual funds. There can be two reasons why our replicating portfolios better explain the out-of-sample variation in individual CSFB/Tremont funds. First, CSFB/Tremont indexes are constructed using a subset of funds and are weighted by assets under management. As a result, they give higher weight to larger funds. In contrast, our analysis of individual funds includes all funds and the mean adjusted R^2 is based on an equally weighted average of all funds. Second, the composition of the CSFB/Tremont indexes may change during the out-of-sample (i.e., post June 2000) period while the composition of the index-replicating portfolios remains the same. These two reasons may lead to the CSFB/Tremont indexes explaining a smaller proportion of out-of-sample variation in individual hedge funds.

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