1 (10 points). Find the domain and the range of the following function

\[ f(x, y, z) = \frac{\sqrt{1-x^2} + \sqrt{y^2-4}}{1 - \sqrt{9-z^2}} \]
2 (15 points). Identify the given surfaces and sketch them

\[ x - y^2 - 6z^2 = 0, \quad \frac{x^2}{4} - \frac{y^2}{9} = 1 \]
3 (15 points). Find $f_x(x, y, z)$ and $f_z(x, y, z)$ by forming the appropriate difference quotient and taking the limit as $h \to 0$. 


4 (20 points). Let $z = x^2 - y^2$ and let $C$ be the curve of intersection of the surface with the plane $y = 3$. Find the equation for the tangent line to the graph of $C$ at the point $(-3, 3, 0)$. 
5 (15 points). Show that

\[
\lim_{{(x,y) \to (0,0)}} \frac{x^2 - y^2}{x^2 + y^2}
\]

does not exist.
6 (20 points). Determine whether a function \( z = f(x, y) \) with the following gradient \( \nabla f(z, y) = (x^2 + y)i + (y^3 + x)j \) may exist. If so, find such a function. Hint: the mixed partials test.
7 (30 points). Find the directional derivative of \( f(x, y, z) = x^2 + yz \) at \((1, -3, 2)\) in the direction of the path \( \mathbf{r}(t) = t^2 \mathbf{i} + 3t \mathbf{j} + (1 - t^3) \mathbf{k} \). Hint: firstly, find \( \mathbf{r}'(t) \) at the moment of \( t =? \) when \( \mathbf{r}(t) \) goes through the given point.
8 (25 points). The radius $r$ of a right circular cylinder decreases at the rate of 2 centimeters per second. At what rate the height $h$ of the cylinder should change in order for its volume not to change at the instant when $r = 10$ and $h = 5$. 
9 (30 points). Let \( f(x, y) = x^2 + y^2 - 1 \) be \( C^1 \) everywhere. Let \( a(0, 0) \) and \( b(1, 1) \). Find the point \( c \) on the line segment connecting \( a \) and \( b \) where \( f(b) - f(a) = \nabla f(c) \cdot (b - a) \).
10 (Bonus 15 points). Let $f(x, y) = 2xy + \sin(xe^y)$. Find $dy/dx$. 
11 (20 points). Show that the sphere $x^2 + y^2 + z^2 - 8x - 8y - 6z + 24$ is tangent to the ellipsoid $x^2 + 3y^2 + 2z^2 = 9$ at the point $(2, 1, 1)$. Find the equations of the tangents planes at this point. What is the equation of the normal line at the point of tangency?