Georgia State University
(This paper consists of 11 pages.)

Final Exam

May 3, 2001

Last name: ____________________________________________
First name: ____________________________________________

Points

Show all of your work. Calculators are not needed or permitted. Write neatly. Place answers in the space provided.

(10 pts) Show that the vectors $\overrightarrow{AB} = (1, 1, -1)$, $\overrightarrow{AC} = (2, 3, -2)$ and $\overrightarrow{AD} = (4, 5, -4)$ are coplanar.
(10 pts) Find the area of the triangle with the vertices $A(3, 0, -10), B(4, 2, 5), C(7, -2, 4)$. 
(10 pts) Find the distance from the point $C(7, -2, 4)$ to the line through the points $A(3, 0, -10)$ and $B(4, 2, 5)$. 
(20 pts) Find the equation for each of the following planes:

a) Plane containing the point \((2, -1, 3)\) and perpendicular to the line

\[ x = 1 + 3t, \quad y = 4t, \quad z = 2 - t \]

b) Plane containing the points \(P(1, 1, 1), \ Q(2, 3, 3)\) and \(R(1, -1, 2)\).
(25 pts) A surface is represented by the equation \( F(x, y, z) = xy + 2xz^2 + 3yz = 56 \). Find

a. (10 pts) the equation of the plane tangent to this surface at \((2, 1, 3)\);

b. (10 pts) Find the directional derivative of \( F(x, y, z) \) at the point \((2, 1, 3)\) in the direction of \( \mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \);

c. (5 pts) Find \( \frac{\partial z}{\partial y} \) on this surface at \((2, 1, 3)\).
(20 pts) Find and classify the stationary points of the function
\[ f(x, y) = x^3 - xy^2 + 3xy. \]
(25 pts) Use the method of Lagrange multipliers to find the largest value and the smallest value of \( f(x, y, z) = xz + y^2 \) on the sphere \( x^2 + y^2 + z^2 = 4 \).
(30 pts) Sketch the domain the area of which is given by the integral

\[
\int_{0}^{1} \int_{0}^{\sqrt{2-y^2}} dx \, dy.
\]

Change the order of integration and find the area.
(30 pts) Find the volume of the 3-D region enclosed by the surfaces $y = x^2$, $y = 4$, $z = 5 + x$, $z = 2$. 
(20 pts) Find the volume of the solid $T$ that is bounded by the paraboloid $z = 3(x^2 + y^2)$ and the plane $z = 12$ (Hint: use cylindrical coordinates).
Bonus (20 pts) Evaluate the repeated integral by changing to spherical coordinates

\[ \int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} (x^2 + y^2 + z^2) \, dz \, dx \, dy \]