(15 points) An object is moving in three-space according to the parametric equations \( x(t) = \sin(t), \quad y(t) = \cos(t), \quad z(t) = \sin(2t) \). Find

position vector \( \mathbf{r}(t) = \)

velocity vector \( \mathbf{v}(t) = \)

acceleration vector \( \mathbf{a}(t) = \)

speed \( v(t) = \)

equation for the tangent line at the moment \( t = 0 \) \( \mathbf{R}(u) = \)

the unit tangent vector at the moment \( t = 0 \) \( \mathbf{T}(0) = \)
(15 points) Identify the surface and find the traces. Then sketch the surface

\[ x^2 + 4z^2 = 4y \]

\[ 4x^2 + y^2 - z^2 = 0 \]

\[ 4x^2 - y^2 - 4z^2 = 4 \]
(15 points) A quantity $Q$ depends upon $x$ and $y$ according to $Q(x, y) = ye^{-xy}$.

Find the second partials of $Q(x, y)$

Let both $x$ and $y$ be changing with time $t$ and at a certain instant you know that $x = 1$, $y = 2$, $x'(t) = 2$, $y'(t) = -2$.

Use the chain rule to find $Q'(t)$ at this instant.
(15 points) Let \( f(x, y) = 2x\sqrt{x + 2y} \)

Find the gradient vector of \( f(x, y) \) \( \nabla f(x, y) = \)

Find the directional derivative of \( f(x, y) \) at \((x, y) = (1, 4)\) in the direction of the vector \( \mathbf{a} = -\mathbf{i} + 4\mathbf{j} \)

Find a unit vector in the direction in which \( f(x, y) \) decreases most rapidly at \((x, y) = (1, 4)\)

Give the rate of change of \( f(x, y) \) in that direction at \((x, y) = (1, 4)\)
(15 points) Find the length of the given curve

\[ r(t) = ti + \left(\frac{1}{4}t^2 - \frac{1}{2}\ln t\right)j, \quad t \in [1, 5] \]
(15 points) Find \( \lim_{(x,y) \to (0,0)} \frac{x^2y^2}{x^4 + y^4} \) or show that it does not exist.
(10 points) The temperature at \((x, y)\) is given by \(T(x, y) = x^2 + 2x - y\). Sketch the level curves \(T(x, y) = 0\) and \(T(x, y) = -1\).
(10 points) The equation

\[ \frac{\partial T}{\partial t} = \delta \frac{\partial^2 T}{\partial x^2}, \]

where \( \delta \neq 0 \) is some constant, is called the heat or diffusion equation. Verify that the function \( T(t, x) = \frac{1}{\sqrt{t}} e^{-x^2/(4\delta t)} \) satisfies it.
(15 points) Let $f(x, y)$ be a function with everywhere continuous second partials. Is it possible that

$$\frac{\partial f}{\partial x} = y + x^2, \quad \text{and} \quad \frac{\partial f}{\partial y} = x + e^y?$$

If it is, can you reconstruct $f(x, y)$?