

Product

Find the following product.

$$\sqrt{72+\sqrt{72+\sqrt{72+\sqrt{72+\sqrt{\dots}}}}} \cdot \sqrt{56-\sqrt{56-\sqrt{56-\sqrt{56-\sqrt{\dots}}}}} = ?$$

♣ Please **Submit** your solution to

- Dr. Erol Akbas, eakbas@gsu.edu or
- Dr. Tirtha Timsina, ttimsina@gsu.edu

before the deadline: **Monday, October 31st, 7:00PM.**

♣ The **WINNER** will be awarded with a \$25 gift certificate and will be announced in the **NEXT** issue.

Problem of the last month: Real Roots

Let C denotes the set of complex numbers and let $f : C \rightarrow C$ be a function defined

as $f(x) = (x-i)^{10} + (x+i)^{10}$. Show that f has a real zero.

Solution: The fundamental theorem of algebra states that every non-constant single-variable polynomial with complex coefficients has at least one complex root.

$$\text{Let } f(x) = 0 \Leftrightarrow (x-i)^{10} = -(x+i)^{10} \Rightarrow \|(x-i)^{10}\| = \|(x+i)^{10}\| \Leftrightarrow$$

$$\|(x-i)^{10}\| = \|(x+i)^{10}\| \Leftrightarrow \|x-i\|^{10} = \|x+i\|^{10} \Leftrightarrow \|x-i\| = \|x+i\|.$$

Let $x = a+bi$ where a, b are real numbers.

$$\Rightarrow \|(a+bi-i)\| = \|(a+bi+i)\| \Leftrightarrow \|(a+bi-i)\|^2 = \|(a+bi+i)\|^2$$

$$\Leftrightarrow a^2 + (b-1)^2 = a^2 + (b+1)^2 \Leftrightarrow a^2 + b^2 - 2b + 1 = a^2 + b^2 + 2b + 1$$

$$\Leftrightarrow -2b = 2b \Leftrightarrow b = 0$$

\Rightarrow Any zero of the polynomial $f(x) = (x-i)^{10} + (x+i)^{10}$ has to be a real number.

Winner: Daniel Balena

Participants with correct solutions: Daniel Balena, John Hull, Wenyan Zhou