Problem of the Month – April

Let $a_1, a_2, \ldots, a_n, \ldots$ be an infinite sequence with $a_n = \frac{p_n}{q_n}$ for integers p_n and q_n. Show that if $a_n \to L$ for some *irrational* number L (as $n \to \infty$), then $q_n \to \infty$.

Hint: start with the fact that every infinite sequence either goes to ∞ or has a bounded subsequence.

Deadline: April 24, 2009, 5:00pm.

- You may get a copy of this from the wall behind you.
- Submit your solution to Dr. Yi Zhao by *yzhao6@gsu.edu* or drop a hard copy in his mailbox before the deadline.
- This is the last problem of spring. Have a nice summer!

Problem of March:

Problem: Let $[n] = \{1, 2, \ldots, n\}$. Determine the number of pairs (A, B) such that A, B are subsets of $[n]$ and A is a subset of B. (You answer should be a simple function of n instead of a complicated summation.)

Solution: It is easy to verify that the number of subsets A of $[n]$ is 2^n because for each element of $[n]$, there are two possibilities: either it is in A or not. We apply the same idea to count the number of pairs (A, B) such that A, B are subsets of $[n]$ and A is a subset of B: for each element of $[n]$, it is either in A (therefore in B), or in $B \setminus A$ (in B but not in A), or not in B. The answer is 3^n.

Winner: Ben Sirb.