

**Imagine** a planet in the shape of a regular tetrahedron (its surface consists of 4 equilateral triangles). Suppose that on each face there is a car traveling at a constant speed in clockwise direction along the edges bounding the face. Can they travel without crashing?



SUBMIT your solution to

- Dr. Erol Akbas @ matexa@langate.gsu.edu or
- Dr. Yuanhui Xiao @ matyxx@langate.gsu.edu

before the **deadline: Friday, February 26, 2010, 5:00PM.**

You may get a copy of this problem from **the wall behind you.**

## Problem of Last Month: An Equation

Let  $x$  be a real number and  $\text{frac}(x) = x - \lfloor x \rfloor$  (fractional part of  $x$ ) where  $\lfloor x \rfloor$  is the greatest integer function. For example:  $\text{frac}(5.87) = 0.87$  or  $\text{frac}(\frac{7}{5}) = \frac{2}{5}$  or  $\text{frac}(-\frac{1}{3}) = \frac{2}{3}$ .

Find a positive real number  $x$  such that  $\text{frac}(x) + \text{frac}(\frac{1}{x}) = 1$ .

**Winner: Reimbay Reimbayev.**

**Participant(s) with Correct Solution:** Robert Xu, Donald Davis.

**Solution.** We can safely exclude 0 and 1 as solutions. Suppose that  $n = \lfloor 1/x \rfloor$  and  $y = \text{frac}(1/x)$ . If we look for a solution  $x$  in  $(0, 1)$ , then  $\text{frac}(x) = x$ . So,

$$x + y = x + 1/x - n = 1.$$

Doing some algebra would result in the following quadratic equation

$$x^2 - (n + 1)x + 1 = 0,$$

which has the solutions

$$x = \frac{n + 1 \pm \sqrt{(n + 3)(n - 1)}}{2}, \quad n = 2, 3, \dots$$

A solution is

$$\frac{2 + 1 - \sqrt{(2 + 3)(2 - 1)}}{2} = \frac{3 - \sqrt{5}}{2} = 0.381966011\dots$$