

**Part I, Problem 1**

Find the following limit:  $\lim_{x \rightarrow 0} \frac{\sin^2(3x)}{x \tan x}$ .

- (A) 0
- (B) 3
- (C) 9
- (D) Does not exist

**Part I, Problem 2**

Find the following limit:  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right)^{4n}$ .

- (A) 1
- (B)  $e^2$
- (C)  $\sqrt{e}$
- (D) Diverges to  $+\infty$

**Part I, Problem 3**

Find:  $\int \min(x, x^2) dx + \int \max(x, x^2) dx$ .

(A)  $\frac{x^2}{2} + C$

(B)  $\frac{x^3}{3} + C$

(C)  $\frac{x^2}{2} + \frac{x^3}{3} + C$

(D) Cannot be integrated

**Part I, Problem 4**

$$1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = ?$$

(A)  $\frac{n(n+1)(2n+1)}{6}$

(B)  $\frac{n(n+1)(2n-1)}{6}$

(C)  $\frac{n(n-1)(2n+1)}{6}$

(D)  $\frac{n(n-1)(2n-1)}{6}$

**Part I, Problem 5**

Find a value of  $b$  such that the function  $f(x)$  is continuous at  $x = 2$ :

$$f(x) = \begin{cases} \frac{3x^2 - 5x - 2}{x - 2} & \text{if } x \neq 2 \\ b & \text{if } x = 2 \end{cases}$$

- (A) 3
- (B) 5
- (C) 7
- (D) 9

**Part I, Problem 6**

A rectangular plot of land is to be fenced in using two kinds of fencing. Two opposite sides will use heavy duty fencing selling at \$3 per foot, while the remaining two sides will use standard fencing selling for \$2 per foot. What are the dimensions of the rectangular plot of greatest area that can be fenced in at a cost of \$6000?

- (A) 500 ft  $\times$  700 ft
- (B) 500 ft  $\times$  750 ft
- (C) 550 ft  $\times$  700 ft
- (D) 550 ft  $\times$  750 ft

**Part II, Problem 1**

Let  $f(x) = \sin(5x)$ . Find the value of the 2009-th derivative of  $f(x)$  at  $x = 0$ . That is, find  $f^{(2009)}(0)$ .

**Part II, Problem 2**

Let  $f(x) = \sqrt{4 + \sqrt{3 + \sqrt{x}}}$ . Find the value of the derivative of  $f(x)$  at  $x = 1$ .  
That is, find  $f'(1)$ .

**Part II, Problem 3**

At what point(s) is the following function NOT differentiable:

$$g(x) = \begin{cases} -1 - 2x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x \leq 1 \\ x & \text{if } 1 < x \end{cases}$$

**Part II, Problem 4**

How many distinguishable rearrangements of the letters in CONTEST have both vowels first? For example, OECNTST is one such rearrangement, but OCNTSTE is not.

**Part II, Problem 5**

Suppose a cube is inscribed in a sphere of radius 1. Determine the volume of the cube.

**Part II, Problem 6**

Find the values of  $a$  and  $b$  so that the function  $f(x) = x^2 + ax + b$  has a local minimum at the point  $(6, -5)$ .

**Part III, Problem 1**

Ruth picks pairs of distinct numbers from the set  $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$  and writes in her notebook the bigger element of the pair that she chose. After picking all possible pairs (without repeating any), Ruth added all the numbers she wrote. What is the answer that she got?

**Part III, Problem 2**

Suppose that  $a_1, a_2, a_3, \dots, a_n, \dots$  is an increasing sequence of positive integers such that  $a_{n+1} = a_n + a_{n-1}$  for  $n \geq 2$ , and  $a_7 = 100$ . Determine the value of  $a_8$ .