Part I, Problem 1

Let $x, y, z, t$ be positive integers and $\frac{x}{y} < \frac{z}{t}$. Which of the following is true about $\frac{x + z}{y + t}$?

(A) It is always less than $\frac{x}{y}$.

(B) It is always between $\frac{x}{y}$ and $\frac{z}{t}$.

(C) It is always greater than $\frac{z}{t}$.

(D) None of the above.
Part I, Problem 2

\[
\int_{0}^{\frac{\pi}{2}} \frac{1 + \sin x}{2 + \sin x + \cos x} \, dx
\]
equals:

(A) \(\frac{1}{4}\)

(B) \(\frac{\pi}{4}\)

(C) \(\frac{1}{2}\)

(D) \(\frac{\pi}{2}\)
Part I, Problem 3

Consider the unit circle given by $x^2 + y^2 = 1$ and the point $P = (3, 0)$ on the $x$-axis. Find the slopes of the two tangent lines from $P$ to the unit circle.

(A) $\pm 1$

(B) $\pm \frac{1}{2\sqrt{2}}$

(C) $\pm \frac{1}{\sqrt{3}}$

(D) $\pm \frac{1}{3}$
Part I, Problem 4

A rug is to fit in a room so that a border of same width is left on all four sides. If the room is 23 feet by 25 feet and the area of the rug is 288 square feet, how wide will the border be?

(A) 2.5 ft
(B) 3.5 ft
(C) 5 ft
(D) 7 ft
Part I, Problem 5

The value of \( \lim_{x \to 1^+} \left( \frac{x}{x - 1} - \frac{1}{\ln x} \right) \) is:

(A) \( \frac{1}{2} \)

(B) \( \frac{1}{4} \)

(C) 2011

(D) 2
Part I, Problem 6

What is the angle between the two arms of the clock when the time is 3 : 15? Give your answer in degrees.

(A) 0°
(B) 15°
(C) 12°
(D) 7.5°
Part II, Problem 1

\[ \lim_{n \to \infty} (\sqrt{n^8 + 2011n^4 + 1} - n^4) = ? \]
Part II, Problem 2

Find the equation of the tangent line(s) (if there are any) to the curve $y = 3x^2$ such that the tangent line passes through the point $(1, -9)$. Write your equation(s) in the slope-intercept form. (If there are no such lines, state your reason.)
Part II, Problem 3

Prove that for any positive number $p$ the following inequality holds:

$$\frac{p^2 + p + 1}{p} \geq 3.$$
Part II, Problem 4

Prove that if $n \geq 4$ then $\sqrt[3]{3} \geq \sqrt{n}$. 
Part II, Problem 5

Prove that for every integer $n$, 30 divides $n^5 - n$. 
Part II, Problem 6

$ABCD$ and $CEFG$ are squares with sides 4 and 2, respectively. Find the area of the triangle $BDH$. 

\[ \text{Area of } BDH = \]
Part III, Problem 1

Let $m_1$, $m_2$, $m_3$ be the slopes of the three sides of an equilateral triangle. What is the value of $m_1 \cdot m_2 + m_2 \cdot m_3 + m_3 \cdot m_1$?
Part III, Problem 2

Find the dimensions of a rectangle whose sides are integer numbers, if we know that the perimeter of this rectangle equals its area.