Part I, Problem 1

What is the last digit of the number $2013^{2013}$?

(A) 1
(B) 3
(C) 7
(D) 9
(E) can’t be determined
Part I, Problem 2

In how many ways can the eight digits 2, 2, 4, 4, 6, 6, 8, 8 be arranged to form two 4-digit numbers $A$ and $B$ such that $B = 2A$?

(A) 4
(B) 3
(C) 2
(D) 1
(E) 0
Part I, Problem 3

In the triangle $ABC$, $[AE]$ is a bisector, $[DE]$ is parallel to $[AB]$, $5|AB| = 3|AC|$, and $|AD| = 3$. What is $|AB|$?

(A) $\frac{36}{5}$
(B) $\frac{24}{5}$
(C) $\frac{12}{5}$
(D) $\frac{9}{5}$
(E) 5
Part I, Problem 4

Let \( f(x) = x^{\ln(x)} \). Find \( f'(e) \).

(A) 0
(B) 1
(C) 2
(D) \( e \)
(E) \( e^e \)
Part I, Problem 5

Find the following limit: \( \lim_{x \to 1^-} (2 - x)^{\tan(\pi x/2)} \)

(A) 1
(B) \( \frac{\pi}{2} \)
(C) \( \frac{2}{\pi} \)
(D) \( e^{\pi/2} \)
(E) \( e^{2/\pi} \)
Part I, Problem 6

Find the following limit: \( \lim_{x \to 0} \frac{\sin^3 x - \sin^{2013} x}{x^3 - x^{2013}} = ? \)

(A) 0
(B) 1
(C) 3
(D) 2013
(E) does not exist
Part II, Problem 1

Show that a number has an odd number of divisors if and only if it is a complete square.
Part II, Problem 2

Let \( A_n = \sum_{j=1}^{n} \frac{j}{j^4 + j^2 + 1} \). Find \( \lim_{n \to \infty} A_n \). (Hint: first, write the sum in a compact form, so that you can then find the limit.)
Part II, Problem 3

A ladder standing on a smooth floor against a wall starts sliding down on to the floor. Along what curve does a cat sitting in the middle of the ladder move?
Part II, Problem 4

Find the following limit: \( \lim_{n \to \infty} \prod_{k=3}^{n} \left( 1 - \frac{1}{k^2} \right) \).
Part II, Problem 5

The curve \((x^2 + y^2)^2 = x^2 - y^2\) is called a *lemniscate*. The curve is shown below. Find the four points of the curve at which the tangent line to the curve is horizontal.
Part II, Problem 6

Consider the sequence $a_1 = 3$, $a_{n+1} = a_n + \sin(a_n)$ for $n \geq 2$. Show that the sequence converges to $\pi$. 
Part III, Problem 1

In how many ways can the fraction $\frac{1}{100}$ be expressed in the form $\frac{1}{100} = \frac{1}{x} - \frac{1}{y}$ where $x$ and $y$ are positive integers?
Part III, Problem 2

Anne, Barbara, and Carol are the only contestants in a race. Anne started last and during the race she swapped positions with other contestants seven times, ending the race ahead of Barbara. Who won?