Homework-Assignment 3

Due Wednesday September 23. Please staple your assignment and present your solutions carefully.

(1) (5 points)
Prove that is α is a cycle of length l, then $\alpha^l = 1$.

(2) (5 points) Find a complete factorization of

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 1 & 4 & 3 & 7 & 5 & 9 & 8 & 6 \end{pmatrix}$$

Compute the order of the permutation.

(3) (5 points) Let $n \geq 3$ and $\alpha \in S_n$ such that α commutes with every $\beta \in S_n$. Prove that $\alpha = e$.

(4) (5 points)
Compute 4^{300} modulo 7.

(5) (5 points)(for graduate students only)
If $1 < r \leq n$ prove that there are

$$\frac{n(n-1)\cdots(n-r+1)}{r}$$

cycles of length r in S_n.