Homework-Assignment 4 **Name:** _____

Please staple your assingment. Write-up your solution carefully including all the details of the proof. Due Monday October 5.

(1) (5 points)
Check that $\mathbf{R} \setminus \{-1\}$ together with the operation $*$ defined by:

$$a * b := a + b + ab$$

is an abelian group.

(2) (5 points) Let $\sigma \in S_n$ be a cycle of length l. Prove that if $\sigma = \tau_1 \cdots \tau_s$, where τ_i are transpositions, then $s \geq l - 1$.

(3) (5 points) Prove that in S_n there are as many even permutations as odd permutations.

(4) (5 points) Show that there is some positive integer k, such that $f^k = e$ for all $f \in S_n$, where e is the identity function. (k is INDEPENDENT of any f in S_n)

(5) (5 points)(for graduate students only)
A permutation of the form $\sigma = (i \;\; i+1), i < n$ is called adjacency.
Prove that every transposition $(i \;\; j)$ is a product of an odd number of adjacencies.