

Homework-Assignment 5

Name: _____

Please staple your assignment. Write-up your solution carefully including all the details of the proof. Due October 19.

- (1) (5 points) Let $*$ be an operation defined on a set G such that $*$ is associative.
Assume that there exists an element $e \in G$ such that $e * a = a$ for all $a \in G$. Also assume that for all $a \in G$, there exists $b \in G$ such that $b * a = e$.
Prove that $(G, *)$ is a group.
- (2) (5 points) Find all orders of the elements in $U(\mathbb{Z}_{12})$.
- (3) (5 points)
Find all elements x in S_4 such that $x^4 = e$.
- (4) (5 points) If G is a group in which $a^2 = e$ for all $a \in G$, then G is abelian.
- (5) (5 points) (for graduate students only) Show that a group of order 4 must be abelian.