

Homework-Assignment 6 Name: _____

Please staple your assignment. Write-up your solution carefully including all the details of the proof. Due Wednesday, October 28.

- (1) (5 points) If G is finite abelian and a_1, \dots, a_n are all its elements, show that $x = a_1 \cdots a_n$ satisfies $x^2 = e$.
- (2) (5 points)
 - a) Let G be a group. Let $Z(G) = \{g \in G : gx = xg \ \forall x \in G\}$.
Prove that $Z(G)$ is a subgroup of G (it is called the center of G). Find $Z(S_3)$ of S_3 .
 - b) Let G be a group, and fix an element $a \in G$.
Define $C(a) = \{g \in G : ga = ag\}$. Prove that $C(a)$ is a subgroup of G (it is called the centralizer of a in G).
- (3) (5 points) Let a, b be two elements in a group G . Show that a and bab^{-1} have the same order.
- (4) (5 points)

List the elements of the left coset of (132) with respect to A_4 in S_4 . (Remember that A_4 is a subgroup of S_4).
- (5) (5 points) (for graduate students only) Let H be a subgroup of G . Prove that there is a one-to-one correspondence between the set of left cosets of H in G and the set of right cosets of H in G .