Homework-Assignment 7 Name: _________________________________
Write-up your solution carefully including all the details of the proof. Due Wednesday November 11.

(1) (5 points) Prove that if \( f : G \to H \) is a group homomorphism such that \((|G|, |H|) = 1\), then \( f(x) = 1' \) for all \( x \in G \), where \( 1' \) is the identity element of \( H \).

(2) (5 points) Prove that \( f : G \to G \) \( f(a) = a^{-1} \) is a group homomorphism if and only if \( G \) abelian.

(3) (5 points) Let \( H \) be a subgroup of \( G \) of index 2. Prove that \( g^2 \in H \), for all \( g \in G \).

(4) (5 points) Prove that if \( f : G \to G' \) is a homomorphism and let \( a \in G \) such that \( \text{ord}(a) = n \). Prove that \( \text{ord}(f(a)) \) divides \( n \).

(5) (5 points) (for graduate students only) If \( A, B \) are subgroups of \( G \) such that \( b^{-1}Ab \subseteq A \) for all \( b \in B \), then \( AB \) is a subgroup of \( G \).