Write-up your solution carefully including all the details of the proof. Due Thursday, September 9.

Please staple your assignment. All rings are assumed commutative with identity. The letters $k, K$ are reserved for fields.

(1) (5 points) Compute the Jacobson radical of $F[X]$ where $F$ is a finite field.
(2) (5 points) Compute the Jacobson radical of $\mathbb{Z}[X]$.
(3) (5 points) Let $R$ be a Noetherian ring of dimension $d \geq 2$. Prove that there are infinitely many prime ideals of height 1.
(4) (5 points) Let $R$ be a Noetherian ring and $I \leq R$ an ideal of $R$. Let $M$ be a finitely generated $R$-module. Show that there exists a largest submodule $N \subset M$ such that $N$ is annihilated by an element of the form $1 - r, r \in R$. Show that

$$\cap_{n=1}^\infty I^n M = N.$$ 

(5) (5 points) Compute the height of the ideal generated by $t^3, t^4$ in $k[t^3, t^4, t^5]$. 
