Spring 2010 Polynomials Homework-Assignment 3
Selected solutions

(1) (5 points) Compute the center of mass of $1 + i$, $1 - i$, $2 - 3i$, $7i$ with respect to $2i$.

Proof. $\xi(z) = z - 3 \frac{f(z)}{f'(z)}$, so $\xi(i) = i - 3 \frac{1 - 2i}{-4} = i + \frac{3 - 6i}{4} = \frac{3 - 2i}{4}$. □

(2) (5 points) Compute the center of mass of the roots of $x^3 - x + 1$ with respect to $i$.

(3) (5 points) Construct an example of a polynomial of degree $f$ at least 3 such that it has one simple root $z_0$ with the property that $f''(z_0) \neq 0$.

(4) (5 points) Let $f(x) = x^3 - 2x^2 + 1$ and $\gamma$ be a circle that passes through 1 and contains integer 3. Show that at least one of the roots for $f$ is strictly inside $\gamma$.

Proof. Note that $f(1) = 0$ and $f'(1) = -1 \neq 0$ so 1 is a simple root.

Then $X(1) = 1 - 2 \cdot 2 \frac{f'(1)}{f''(1)} = 1 - 4 \frac{-1}{2} = 3$.

If the other roots are both outside our circle, then $X(1) = 3$ will also have to be in the closed domain determined by the exterior of the circle. This contradicts the fact that 3 is said to be inside the circle. □

(5) (5 points)(graduate students)

Use Laguerre’s criterion to show that some of the roots of $f(z) = z^3 - z^2 + z - 1$ are not real.