(1) (5 points) Compute the apolar derivative of \( f(z) = z^4 + 2z + 1 \) with respect to \( \xi = 2i \).

(2) (5 points) Find an apolar polynomial for \( f(z) = z^4 + 3z^2 + 3 \).

(3) (5 points) Determine whether or not \( f(z) = z^3 + 2z + 2 \) and \( g(z) = z^3 + z^2 + 1 \) have a common root.

(4) (5 points) Let \( f(z) = z^3 + 6z^2 - 3z + 3 \). Show that at least one root of \( f \) has absolute value less or equal to 2. (Hint: consider a polynomial \( g = z^3 + z^2 + az \) apolar with \( f \)).

(5) (5 points) (Graduate students) If \( f(z) = z^3 + 3z^2 + 9z + a = 0 \), \( a \in \mathbb{R} \) has a complex nonreal root \( w \), then \( |\text{Im}(w)| > \sqrt{2} \).