Spring 2009 Abstract Algebra II Homework-Assignment 1 **Name: ________________**

Write-up your solution carefully including all the details of the proof. Due Tuesday, January 20.

Please staple your assignment.

(1) (5 points) Let H be a subgroup of G of finite index. Prove that H contains a normal subgroup of G of finite index.

(2) (5 points) Prove that in a group the subset of all elements that have only a finite number of conjugates is a subgroup.

(3) (5 points) If G is a finite group of order n and p is the smallest prime dividing $\mid G \mid$ then any subgroup of index p is normal.

(4) (5 points) If the center of G has index n, prove that every conjugacy class has at most n elements.

(5) (5 points) let $n \geq 4$, $\sigma = (12)(34)$. Show that the cardinality of the normalizer of σ in S_n is $8 \cdot (n-4)!$.