Spring 2009 Abstract Algebra II Homework-Assignment 3   **Name:** ______________

Write-up your solution carefully including all the details of the proof. Due Tuesday, February 10.

Please staple your assignment.

(1) (5 points) Show that any finitely generated subgroup of $\mathbf{Q}$ is cyclic.
(2) (5 points) Let P be a Sylow p-subgroup of G. Then p cannot divide $\mid N(P)/P \mid$.
(3) (5 points) Show that a group of order 1986 is not simple.
(4) (5 points) Show that there is no group G such that $\mid G/Z(G) \mid = 15$.
(5) (5 points) Let M be an additive group. Show that there is only one way to make M a $\mathbf{Z}$-module.

These three problems will count as extra-credit. Each of them is worth 2 points.

(6) Show that $\mathbf{Q}$ cannot be written as the internal direct sum of two abelian subgroups.
(7) Prove that if G is nilpotent and H a proper subgrop G, then H is strictly contained in $N(H)$.
(8) Find all three Sylow subgroups of A_4.