

Write-up your solution carefully including all the details of the proof. Due Thursday, February 19.

Please staple your assignment. All rings are considered with unity.

- (1) (5 points) Let R be a ring and M be a left R -module and $m \in M$. Prove that $\text{Ann}(M) = \{r : rM = 0\}$ is a two-sided ideal and $\text{Ann}(m) = \{r : r \cdot m = 0\}$ is a left ideal in R .
- (2) (5 points) Let M be a left R -module. Compute $\text{Hom}_R(R, M)$. Compute $\text{Hom}_{\mathbf{Z}}(\mathbf{Z}^2, \mathbf{Z})$.
- (3) (5 points) Compute $\text{Hom}_{\mathbf{Z}}(\mathbf{Z}_m, \mathbf{Z}_n)$.
- (4) (5 points) Let K, N, L be R -modules such that $K \oplus N = K \oplus L$ as R -modules implies $N \simeq L$.
- (5) (5 points) Let I be a left ideal of a ring R such that R/I is isomorphic to R as R -module. Show that $I = Re$ where e is idempotent (i.e., $e^2 = e$).

These three problems will count as extra-credit. Each of them is worth 2 points.

- (6) Show that \mathbf{Q} is isomorphic to $\text{End}_{\mathbf{Z}}(\mathbf{Q})$ as rings.
- (7) Prove that \mathbf{Q} is not free as \mathbf{Z} -modules.
- (8) Let R be a ring. Prove that if R is completely reducible R -module then every R -module is completely reducible.