$k$-critical structures in $k$-Ore graphs

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Given two graphs $G_1, G_2$ with edges $x_1y_1 \in E(G_1)$ and $x_2y_2 \in E(G_2)$ the Hajós construction forms a new graph $G$ by deleting $x_1y_1$ and $x_2y_2$, identifying $x_1$ and $x_2$ into a single vertex, and adding an edge $y_1y_2$. Starting with the complete graph $K_k$ and using repeated applications of the Hajós construction, we can construct and infinite family of $k$-vertex-critical graphs; that is, graphs which are not $(k - 1)$-colorable, but for which every proper subgraph is $(k - 1)$-colorable.

The Ore construction is a generalization of the Hajós construction. The graphs constructible from $K_k$ and repeated applications of the Ore construction are called $k$-Ore graphs; these graphs play an important role in answering Dirac’s question of the minimum number of edges in a $k$-critical graph on $n$ vertices.

Kostochka and Yancey recently established a bound $|E(G)| \geq \left\lceil \frac{(k+1)(k-2)n-k(k-3)}{2(k-1)} \right\rceil$ for $k$-critical graphs on $n$ vertices, and the $k$-Ore graphs show that this bound is tight. In this talk, we examine more closely some of the properties of $k$-Ore graphs, which will be helpful when extending the Kostochka and Yancey result. In particular, we define and bound the number of $k$-critical structures in $k$-Ore graphs.