$K_{s,t}$ minors in $(s+t)$-chromatic graphs
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In search of ways to attack Hadwiger’s Conjecture, Woodall and independently Seymour suggested to prove the weaker conjecture that

Every $(s+t)$-chromatic graph has a $K_{s,t}$-minor.

If the conjecture were true for all values of $s$ and $t$, it would imply that for $k \geq 2$ every $(2k-2)$-chromatic graph has a $K_k$-minor. The conjecture is evident for $s = 1$. The validity of the conjecture for $s = 2$ and all $t$ was proved by Woodall, and also follows from a result by Chudnovsky, Reed, and Seymour. Prince and the speaker proved the conjecture for $s = 3$ and $t \geq 6500$. The aim of the talk is to show that for every fixed $s$ and large $t$, the conjecture holds in a slightly stronger form:

Let $s$ and $t$ be positive integers such that $t > t_0(s) := (240s \log_2 s)\log_2 s + 1$. Then every $(s+t)$-chromatic graph has a $K_{s,t}^*$-minor, where $K_{s,t}^*$ is obtained from $K_{s,t}$ by adding all edges between the vertices of the partite set of size $s$.

The result is sharp in the sense that for every $s, t \geq 3$, there are infinitely many $(s+t)$-critical graphs that do not have $K_{s,t+1}$-minors.