Extremal graph theory and the signless Laplacian

Vladimir Nikiforov
University of Memphis

Given a graph $G$, write $A$ for its adjacency matrix and let $D$ be the diagonal matrix of the row-sums of $A$, that is to say, the degrees of $G$. The matrix $Q = A + D$, called the signless Laplacian or the $Q$-matrix of $G$, has received a lot of attention recently. This talk will present results about $q(G)$, the largest eigenvalue of the signless Laplacian of a graph $G$, when $G$ lacks certain subgraphs.

More precisely, the general extremal problem that will be discussed is the following one: *How large can $q(G)$ be, if $G$ is a graph of order $n$, with no subgraph isomorphic to a given graph $F$?*

When the chromatic number of $F$ is at least 3, a tight asymptotic solution to this problem was given fairly recently. However, for bipartite graphs the problem is subtler and requires different methods. The reason is that if $F$ is a nontrivial bipartite graph and $G$ is a graph of order $n$ with no subgraph isomorphic to $F$, then $q(G) \leq n + o(n)$, and this bound is tight.

This talk will focus on new results when $F$ is an even cycle or a path. In particular, if $G$ is a graph of order $n$, with no 4-cycle, then

$$q(G) \leq \frac{n + 2 + \sqrt{n^2 - 4n + 12}}{2}.$$  

This bound is tight since equality holds for the friendship graph.