Supereulerian graphs with width $s$ and $s$-collapsible graphs

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Abstract: For an integer $s \geq 0$ and for $u, v \in V(G)$ with $u \neq v$, an $(s; u, v)$-trail-system of $G$ is a subgraph $H$ consisting of $s$ edge-disjoint $(u, v)$-trails. A graph is supereulerian with width $s$ if $\forall u, v \in V(G)$ with $u \neq v$, $G$ has a spanning $(s; u, v)$-trail-system. The supereulerian width $\mu'(G)$ of a graph $G$ is the largest integer $k$ such that $G$ is supereulerian with width $k$ for any integer $s$ with $1 \leq s \leq k$. Thus a graph $G$ with $\mu'(G) \geq 2$ has a spanning Eulerian subgraph. (Such graphs are called supereulerian graphs in the literature). Catlin introduced collapsible graphs together with a reduction method to study conditions for a graph to be supereulerian, and showed that every collapsible graph $G$ satisfies $\mu'(G) \geq 2$. In this talk, we will introduce a generalization of Catlin’s reduction method and who applications to the problems of determining the supereulerian width of graphs and of determining some of the $H$-linked problems in line graphs.