Note on upper density of quasi-random hypergraphs

Abstract

In 1964, Erdős proved that for any \( \alpha > 0 \), an \( l \)-uniform hypergraph \( G \) with \( n \geq n_0(\alpha, l) \) vertices and \( \alpha(\binom{n}{l}) \) edges contains a large complete \( l \)-equipartite subgraph. This implies that any sufficiently large \( G \) with density \( \alpha > 0 \) contains a large subgraph with density at least \( l! / l^l \).

In this talk we discuss a similar problem for \( l \)-uniform hypergraphs \( Q \) with a (weak) quasi-random property. We prove any sufficiently large quasi-random \( l \)-uniform hypergraph \( Q \) with density \( \alpha > 0 \) contains a large subgraph with density at least \( \frac{(l-1)!}{l!} \). In particular, for \( l = 3 \), any sufficiently large such \( Q \) contains a large subgraph with density at least \( \frac{1}{4} \) which is the best possible lower bound.

This is joint work with Vojtěch Rödl.