Math 615 – Winter 2005
1:00–2:00pm @ 3088 East Hall, Monday, Wednesday and Friday

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Office Hours: 1:00–3:00pm Tuesday and Thursday at 1851 East Hall. Or by appointment.
Or by e-mail.

Textbook: Optional. The following sources, among others, should be helpful:

• M. Hochster’s Math 715 lecture notes
  http://www.math.lsa.umich.edu/~hochster/715/715.html
• M. Hochster and C. Huneke, Tight closure, invariant theory, and the Briançon-
• C. Huneke, Tight closure and its applications, CMBS regional Conference Series
  in Mathematics, no. 88.

Homework: There are about 5–6 homework assignments.

Syllabus: Depending on the interests of the students and the time available, this course
will cover topics as follows:

• Tight closure theory. We start by defining the notion of tight closure over rings
  of characteristic $p > 0$ as well as studying its properties. We will also see how
  tight closure is used to produce theorems that are hard to prove otherwise (e.g.,
  the Briançon-Skoda theorem in equal characteristic case).
• The theory of Hilbert-Kunz multiplicity. Hilbert-Kunz multiplicity is defined over
  rings of characteristic $p > 0$ and it is closely related to the tight closure theory. In
  particular, we will show that an unmixed local ring with Hilbert-Kunz multiplicity
  equal to 1 (equivalently, sufficiently close to 1) is regular.
• The module structure of $R$ over its subring $R^p$ in the case of characteristic $p > 0$.
  Topics include rings of finite F-representation type, the theory of F-signature, etc.
  In particular, a local ring with positive F-signature is strongly F-regular. If the
  F-signature is equal to 1 (equivalently, sufficiently close to 1), then the ring is
  regular.
• Integral closures of ideals. We will start from the definition of integral closure
  and then proceed to study some of its subtle properties. In particular, we aim to
  present detailed versions of Briançon-Skoda type theorems. Time permitting, we
  may treat the Lipman-Sathaye Jacobian theorem.

A strong effort will be made to accommodate students with different backgrounds.

Website: http://www.math.lsa.umich.edu/~ywyao/2005W/math615.html