Discounted Cost Optimization in Make-to-Stock Production-Inventory Systems with Continuous Replenishment Under the Base-Stock/Backorders/Lost-Sales Policy

Junmin Shi

jshi@pegasus.rutgers.edu

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Abstract

Consider a continuous-review single-product Make-to-Stock (MTS) production-inventory system with continuous replenishment under the Base-Stock/Backorders/Lost-Sales policy. In this system, the inventory level may assume both positive values and negative values, where the latter represent the inventory position in the presence of backorders. The demand process consists of mutually independent iid interarrival times and iid demand sizes. Replenishment is continuous and governed by the base-stock policy as follows: it proceeds at a constant rate while the inventory level is below the base-stock level, and is suspended, otherwise. Inventory depletion is governed by a combination of backorders and lost sales, using a lost-sales threshold as follows: if a demand fulfillment causes an inventory shortage (that is, the inventory level becomes negative), then the shortage up to the lost-sales threshold is backordered, and the rest of the shortage (referred to as the lost-sales size) represents a (fractional) lost sale.

The system incurs two types of costs, depending on its operational mode. In normal mode, the inventory level is non-negative, and a carrying cost is incurred at a constant rate (holding cost). However, in underage mode, the inventory level becomes negative, and an underage cost consisting of two components is incurred per stockout event as follows: a fixed stockout penalty component is always assessed, while a lost-sales penalty component is assessed proportionally to the lost-sales size (if any). The cost function under study is the expected discounted cost over an infinite time horizon, given the initial inventory level.

The goal of this paper is to optimize the expected discounted cost over an infinite time horizon with respect to the base-stock level and replenishment rate parameters. To this end, we derive the expected discounted total cost function via a renewal approach. We then proceed to derive the partial derivatives with respect to the aforementioned parameters, and use them to obtain the corresponding optimal minimal-cost solution.

Keywords and Phrases: Base-Stock/Backorders/Lost-Sales policy, Continuous replenishment, Expected discounted cost Lost-Sales, Make to Stock, Stockout.