A Rational Expectations Model for Commodity Prices when Both Buyer and Seller Have Inventory Buffers

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Abstract

1 Introduction

Methodologically, we use similar techniques as have been employed by Wu and Chen [3]. Model-wise, we note that the focus of Wu and Chen was one aggregate middle-stream decision maker, who has to deal with both upper-stream supply and down-stream demand, which in turn respond to changes in, respectively, raw material and finished product prices in given fashions. We model two aggregate decision makers, one aggregate buyer and one aggregate seller. To achieve market clearance, we match optimal sales and purchase rates of the two parties, rather than as done in Wu and Chen, match exogenous supply with optimal purchase and exogenous demand with optimal sales. In Routledge, Seppi, and Spatt [2], inventory traders hold stocks.

2 Formulation

We suppose $w_t$ is an $n$-dimensional standard Brownian motion. Let $k_t$ be the $m$-dimensional state of the world that is governed by the following stochastic equation:

$$dk_t = \mu_0(k_t) \cdot dt + \Sigma_0^T(k_t) \cdot dw_t,$$

where $\mu_0(\cdot)$ is an $m$-dimensional vector-valued function, and $\Sigma_0(\cdot)$ is an $n \times m$-dimensional matrix-valued function. We also assume that there is a discount rate $r(k_t)$, so that the cumulative discount is $R_t = \int_0^t r(k_u) \cdot du$. 