NON-PARAMETRIC TESTS OF THE TRAGEDY OF THE COMMONS*

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Abstract

Drawing on recent results in the industrial organization literature (Carvajal et al. 2013), we derive non-parametric tests of the tragedy of the commons. Our approach allows group output to be any arbitrarily concave, differentiable function of total inputs and for individual exploiters of the resource to have any arbitrary convex, differentiable cost of supplying inputs. We show that observed group output and individual inputs are consistent with the tragedy of the commons only if the data follow certain patterns denoted as the common-ratio property and the co-monotone property. Our basic results for a static average-return game can be extended to tragedies with consistent conjecture (as opposed to Nash) equilibria, to dynamic resources, and to the average-cost game.

JEL Codes:  C14, C72, D21, D23, Q2, Q3

1. Introduction

The "tragedy of the commons" is a classic example of a situation in which strategic incentives, unchecked by property rights or other institutional arrangements, undermine the potential value of a commonly held resource (Hardin 1968). Because individuals do not bear the full decline in marginal productivity when they utilize the resource, they have an incentive to use it too intensively, relative to the group's welfare. In the standard model, individuals receive a prorated share of collective output, proportionate to their inputs, so by increasing inputs they can obtain a larger share of the pie (Gordon 1954, Weitzman 1974, Dasgupta and Heal 1979).¹ Classic examples include sending cattle to a common pasture, fishing from the sea, or extracting oil from a common pool.

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¹ Compare also Sen's (1966) model of cooperatives.
Though examples of the tragedy at work are pervasive, groups can avoid the trap of open-access and devise ways to cooperate and limit access to the commons, effectively managing common-pool resources to avoid the tragedy (Ciriacy-Wantrup and Bishop 1975 and Ostrom 1990). Evidence from laboratory experiments suggests that when they make decisions anonymously and without communication, individuals do over-exploit common resources, producing the "tragedy," but when they can communicate and/or can build other institutions to change incentives, they can overcome the tragedy (Ostrom 2009).

Surprisingly, though, there have been few empirical tests of the standard model with naturally occurring data. Kirkley, Paul, and Squires (2002) and Felthoven, Horrace, and Schnier (2009) outline approaches for measuring capacity utilization in an industry exploiting a common pool resource, such as a fishery, interpreting excess capacity as a symptom of applying excessive variable inputs to the resource. This approach requires estimating a production function for firms. More importantly, it does not provide an explicit mapping from the capacity utilization measure to the strategic behavior in the commons model.

On the other hand, Huang and Smith (2014) have conducted the first micro-level empirical investigation of strategic behavior in a common pool. They develop a dynamic structural model of the microeconomic behavior of fishers operating in an open access fishery. Each fisher chooses his effort to maximize his expected utility given all other fishers' actions, with agglomeration or congestion effect specified such that individual catch per day is affected by the total number of vessels fishing on that day. With estimates from their parametric model, potential efficiency gains can be quantified by comparing the optimal number of vessels to the vessel numbers predicted from the individual maximization problem. However, their approach presupposes Nash behavior in a commons game rather than providing a way to test for such behavior. Moreover, their approach is highly parametric, which has the advantage of allowing for counter-factual policy simulations and welfare analyses, but comes at the cost of bringing in numerous maintained assumptions when it comes to testing for particular modes of strategic behavior.

Drawing on recent work by Carvajal et al. (2013), we develop a non-parametric revealed preference test for the "tragedy" in common pool resources. Carvajal et al. developed a revealed preference test for a Cournot equilibrium, deriving properties that hold when firms are
strategically interacting as predicted by the Cournot model. As the tragedy of the commons and the Cournot model are essentially isomorphic, we can derive similar properties that should hold under the strategic interactions of the tragedy of the commons. This approach has the advantage of requiring no parametric assumptions about production functions or cost functions (beyond convexity) or, in dynamic settings, about the evolution of stocks. The test is derived from the key characteristics of the tragedy of the commons that each agent maximizes its objective function independently and from the proportionate sharing rule. The test can be implemented with panel data of individual inputs and total output. In particular, given panel data on each agent's input and the total output from exploitation, we show that a data set is consistent to the tragedy of the commons with convex cost functions if and only if there is a solution to a linear program that we can explicitly construct from the data. Accordingly, the tests we derive can be applied to various settings with common pool resources.

These notes are organized as follows. In section 2, we present the theoretical results derived for the classic static model of the average return game, in which agents select their inputs and each unit of input receives the average return (rather than marginal return). In section 3, we extend the equilibrium in the static case to incorporate conjectural variation. In section 4, we consider a case with a dynamic resource. In Section 5, we consider the average-cost game as an alternative to the average-return game.

2. A Nonparametric Test of the Tragedy of the Commons: the Static Case

Consider an industry consisting of $I$ profit-maximizing firms, indexed by $i = 1, 2, \ldots, I$, each having free access to an exogenously fixed common property resource, such as a fishing ground. There are $T$ decision periods indexed by $t = 1, 2, \ldots, T$. Denote $q_{it}$ as the extraction effort by firm $i$ in period $t$. For example, $q_{it}$ might be the number of fishing vessels sent forth by firm $i$ on day $t$, or the number of vessel-days in year $t$. Let $Q_t = \sum q_{it}$ be the total level of effort applied to the resource at time $t$. The differentiable production function for the industry at time $t$ is $Y_t = F_t(Q_t)$, with $F(0) = 0$, $F'(Q) > 0$, and $F'$ non-increasing for all $t$. Following the standard commons model (Gordon 1954, Weitzman 1974, Dasgupta and Heal 1979, Cornes and Sandler 1996), each firm's catch is proportionate to its share of total extraction effort. Normalizing output prices to 1, firm $i$'s revenue in period $t$ is $\frac{q_{it}}{Q_t} * F_t(Q_t)$. This assumption captures the characteristic of open-access resources that factors tend to receive their average rather than the
marginal product. Finally, let $C(q_{it})$ denote firm $i$'s cost function, which is a differentiable and non-decreasing function of $q$ and which—for now—we treat as time invariant.

Following Carvajal et al.'s logic for Cournot competition, we say a panel data set $O = \{F_{t, (q_{it})_{i \in I}}\}_{t=1}^{T}$ is consistent with the tragedy of the commons if there exist cost functions $\bar{C}_i$ for each firm $i$, and a concave production functions $\bar{F}_t$ for each observation $t$ which jointly satisfy the following two conditions:

(i) $\bar{F}_t(Q_t) = F_t$

(ii) $q_{i,t} \in \arg\max \bar{q}_{it} \geq 0 \left\{ \frac{\bar{q}_{it}}{Q_t} F_t(Q_t) - \bar{C}_i(\bar{q}_{it}) \right\}$

Condition (i) says the production function must be consistent with observed output at time $t$. Condition (ii) says firm $i$'s input at time $t$ maximizes its profit given the inputs of all other firms (a standard Nash assumption).

It is worth pointing out that we do not need to estimate the production function as do Kirkly, Paul, and Squires (2002) or Huang and Smith (2014). We allow the analyst to explain the data using any production function, as long as it passes through the observed total output and inputs $F_t(Q_t)$ at each decision period. No restrictions are placed on the production function apart from requiring it to be concave. Similarly, no restrictions are placed on firms' cost functions except that they are increasing and convex. These assumptions are sufficient to guarantee a quasi-concave profit function to be maximized.

To see this, note that firm $i$'s profit-maximization problem at time $t$ is:

$$\max_{q_{i,t}} \left\{ \frac{q_{it}}{Q_t} F_t(Q_t) - C_i(q_{i,t}) \right\}$$

Taking other firms' actions as given, the first-order condition is:

$$\frac{q_{it}}{Q_t} F_t'(Q_t) + \left( 1 - \frac{q_{it}}{Q_t} \right) \frac{F_t(Q_t)}{Q_t} = C_i'_{i,t}.$$

This is the standard result that firms equate marginal cost to a weighted average of marginal returns and average returns (Weitzman 1974, Dasgupta and Heal 1979). In the case of a
monopolist, \( q_{i,t} = Q_t \) and the entire weight is on the efficient condition to equate marginal cost to marginal return. In the limit, as the firm grows small, \( q_{i,t}/Q_t \) goes to zero and the firms equate marginal cost to average revenue, thus depleting all resource rents.

Rearranging terms, we obtain:

\[
\frac{F_t(Q_t) - Q_tC'_{i,t}}{q_{i,t}} = \frac{F_t(Q_t)}{Q_t} - F'_t(Q_t).
\]

Notice in Equation (3) that the left hand side involves functions of firm-specific data (inputs \( q_{i,t} \) and marginal costs \( C'_{i,t} \)) while the right-hand side involves only market-wide data (total production \( F \) and marginal product as well as total input \( Q \)). Consequently, from the first-order condition, we obtain a common ratio property comparable to that in Carvajal et al.:

\[
\frac{F_t(Q_t) - Q_tC'_{i,t}}{q_{i,t}} = \frac{F_t(Q_t) - Q_tC'_{j,t}}{q_{j,t}} = \ldots = \frac{F_t(Q_t) - Q_tC'_{i,t}}{q_{i,t}} \geq 0 \text{ for all } t.
\]

In other words, in each time period, functions of firms’ extraction effort and marginal costs should all be equal to one another. The expressions are nonnegative given the concavity of production function. Moreover, because each firm’s cost function is convex, the array \( \{C'_{i,t}\} \) displays increasing marginal costs for each firm \( i \). Thus, if the cost function is time-invariant, we also have the co-monotone property as described in Carvajal et al., such that for all \( i \),

\[ q_{i,t} > q_{i,t'} \rightarrow C'_{i,t} \geq C'_{i,t'}. \]

The main result of this section is that a set of observations is consistent with the tragedy of the commons with convex cost functions if and only if there exist nonnegative numbers \( \{C'_{i,t}\} \) for all \( i,t \) that obey the common ratio and co-monotone properties. In Example 1, we show that certain data sets are not consistent with the tragedy of the commons with the interplay of the two properties.

**Example 1**: Consider the following observations of two firms \( i \) and \( j \) sharing a common-pool resource:

(i) At observation \( t \), \( F_t(Q_t) = 50 \) tons, \( q_{i,t} = 50 \), \( q_{j,t} = 100 \).
(ii) At observation $t'$, $F_{t'}'(Q_{t'}) = 350$ tons, $q_{i,t'} = 70$, $q_{j,t'} = 60$.

Re-arranging the common-ratio property at $t'$ to isolate $C_{j,t'}'$ and using the fact that $rac{q_{j,t'}'}{q_{i,t'}} C_{i,t'}' \geq 0$, we have:

$$C_{j,t'}' = \frac{F_{t'}'(Q_{t'})}{Q_{t'}} - \frac{q_{j,t'}'}{q_{i,t'}} C_{i,t'}' \geq \frac{F_{t'}'(Q_{t'})}{Q_{t'}} - \frac{q_{j,t'}'}{q_{i,t'}} C_{i,t'}' = 0.385.$$  

Now, we know from the first-order condition (2) that $C_{i,t} < \frac{F_t(Q_t)}{Q_t}$, at each time $t$ for all $i$, because $C_{i,t} = \frac{q_{i,t}}{Q_t} \left( F_t'(Q_t) - \frac{F_t(Q_t)}{Q_t} \right) + \frac{F_t(Q_t)}{Q_t}$ and $F_t'(Q_t) - \frac{F_t(Q_t)}{Q_t} < 0$ given the concavity of production function. Thus, $C_{j,t} < \frac{F_t(Q_t)}{Q_t} = 0.33$. In addition, from the co-monotone property, we have $C_{j,t'} \leq C_{i,t}$ because $q_{j,t'} < q_{j,t}$. Thus, in sum, $0.385 \leq C_{j,t'} < C_{j,t} < 0.33$, which is clearly a contradiction. Thus, the data in Example 1 are not consistent with the tragedy of the commons.

As shown by Carvajal et al. (2013), if the data are consistent with the tragedy of the commons, the cost functions can be set identified by finding the set of values that are consistent. Carvajal et al. further show that the assumption of time-invariant cost functions embedded in the co-monotone property can be generalized to allow for common-cost shocks for all firms, such that marginal costs increase or decrease between $t$ and $t'$ by the same additive factor for all firms.

3. The Static Case with Conjectural Variations

Standard models of the tragedy of the commons focus on Nash equilibria. A possible generalization of Nash is a model of conjectural variation, in which firms, rather than take others' actions as given, assume other firms will respond to their own actions. A conjectural variation of zero implies no response, as in the standard Nash model. With non-zero conjectural variations, an equilibrium with consistent conjectures is a point at which firms correctly predict the actions of one another given, in turn, their reactions. Cornes and Sandler (1983) introduce a model of conjectural variations into the commons model. They show that an equilibrium with consistent conjectures is even more pessimistic than the Nash equilibrium, with all rents to the resource dissipated even with only two firms.

As discussed by Carvajal et al. (2013) in the context of oligopoly, the above tests of the tragedy of the commons can be generalized to conjectural variations, but this generalization
naturally weakens the statements can be made (i.e., broadens the set of possible models consistent with the data). Denote $\widetilde{Q}_{i,t}^e$ as the expected input of all other firms at time $t$ given firm $i$'s input $q_{i,t}$. Thus $\sigma_{i,t} = \frac{d\widetilde{Q}_{i,t}^e}{dq_{i,t}}$ represents the other firms' (anticipated) responsiveness to firm $i$'s input of exploitation from the commons, or firm $i$'s influential power. When $\sigma=0$, firms act as if they have no effect on others' effort, which is the Nash assumption. When $\sigma=-1$, firms act as if they have no effect on total effort.\(^2\)

With this generalized model, firm $i$'s maximization problem at time $t$ becomes:

\[
\max_{q_{i,t}} \left\{ \frac{q_{i,t}}{q_{i,t} + \widetilde{Q}_{i,t}^e} \cdot F_t(q_{i,t} + \widetilde{Q}_{i,t}^e) - C_{i,t}(q_{i,t}) \right\}.
\]

The first-order condition is

\[
\frac{F_t(Q_t)}{Q_t} - \left( 1 + \sigma_{i,t} \right) \cdot \frac{q_{i,t}}{Q_t} \cdot \left[ F_t(q_{i,t}) - F_t'(Q_t) \right] = C_{i,t}',
\]

which collapse to Equation (2) when $\sigma_i = 0$. When $\sigma_i = -1$, firms act as if they have no effect on total inputs, and firms equate marginal cost to average return, which is the limiting case of Gordon (1954). Rearranging terms:

\[
\frac{F(Q_t) - Q_t c_{i,t}'}{(1 + \sigma_{i,t}) q_{i,t}} = \frac{F_t(Q_t)}{Q_t} - F_t'(Q_t),
\]

which respectively collapses to Equation (3) when $\sigma_i = 0$.

From the first-order condition, we obtain the $\sigma$-common ratio property (i.e., the common ratio property with conjectural variations):

\[
\frac{F_t(Q_t) - Q_t c_{i,t}'}{(1 + \sigma_i) q_{i,t}} = \frac{F_t(Q_t) - Q_t c_{j,t}'}{(1 + \sigma_j) q_{j,t}} = \cdots = \frac{F_t(Q_t) - Q_t c_{n,t}'}{(1 + \sigma_n) q_{n,t}} \geq 0 \text{ for all } t.
\]

The data set $\mathcal{O}$ is consistent with the $\sigma$-tragedy (i.e., tragedy of the commons with conjectural variation) if there exist cost functions $\overline{C}_{i,t}$ for each firm $i$, and a concave production function $\overline{F}_t$.

\(^2\) Cornes and Sandler (1983) show that $\sigma = -1$ is the only value for which a consistent conjectures equilibrium exists.
at each decision period such that \( \bar{F}_t(Q_t) = F_t \) and \( q_{i,t} \) constitutes a solution to the independent maximization problem with conjectural variation. Slightly different from the previous case without conjectural variation, the main result here is that a set of observations is consistent with the \( \sigma \)-tragedy with convex cost functions if and only if there exist nonnegative numbers \( \{C_{i,t}'\} \) that obey the \( \sigma \)-common ratio property and the co-monotone properties.

It is worth pointing out that any cost functions and production functions that satisfies the conditions in Section 3 also satisfies conditions in Section 2 with symmetric influential power of firms. That is, when \( \sigma_{i,t} = \sigma_{j,t} \) for all \( i,j \), condition (8) becomes the same as condition (4). As discussed by Carvajal et al, this leads to an identification problem: condition (8) cannot be used to test the absolute level of production power in the commons. However, it can be used to test relative power. Generalized to conjectural variations, the results we developed in Section 2 can be interpreted as a test of the symmetry of influential power as measured by \( \sigma_{i,t} \). That is, when a data set passes the test in Section 2, it is consistent with the tragedy of the commons, but it is also consistent with the \( \sigma \)-tragedy in which \( \sigma_{i,t} = \sigma_{j,t} \) for all \( i,j \) (i.e., in which all firms hold symmetric power). Conversely, when a data set fails the test in Section 2, then it is a strong indication that all levels of symmetry influential power have been rejected.

4. The Dynamic Case

Many commons problems take place in the context of a dynamic resource. For example, extraction of oil in a common pool involves depleting the common stock over time; fisheries can be over-harvested but also grow back and some rate depending on the existing stock. In such dynamic settings, we can characterize firm \( i \)'s problem as choosing a path of effort \( q_i(t) \) as follows:

\[
(9) \quad \max_{q_i(t)} \int_0^T \left[ \frac{q_i(t)}{Q(t)} * F(Q(t), s(t)) - C_i(q_i(t)) \right] e^{-rt} dt
\]

Subject to

\[
(10) \quad \dot{s}(t) = N(s(t)) - F(Q(t), s(t));
\]

and
The production function $F$ is now also an increasing function of the stock of the common pool resource $s(t)$ as well as total effort. $\dot{s}$ is the change in the stock, which is a function of the natural growth of the resource $N$, which depends on stock level at each time point, and extraction $F$. In the case of a depletable resource like oil, $N(s)=0$; in the case of a fishery, $N$ represents population growth (and can in principle be negative if predation exceeds reproduction). $r$ is the discount rate. Finally, $K$ is the initial amount of stock of the resource.

In a continuous time setting, a firm's current value Hamiltonian is:

$$
H = \frac{q_i(t)}{Q(t)} * F(Q(t), s(t)) - C_i(q_i(t)) + \mu_{i,t} \left( N(s(t)) - F(Q(t), s(t)) \right),
$$

which is the same as the static case except for the treatment of the stock, which each firm weights by its in-situ value $\mu_{i,t}$. We can define $\mu_t = \sum_i \mu_{i,t}$, where $\mu_t$ is the shadow value of an additional stock to all firms, following the Samuelson rule. If $q_i(t)$ is continuous in $t$, then at each instant firms weight the stock by their share of the total extraction, so $\mu_{i,t} = \frac{q_i(t)}{Q(t)} \mu_t$.

The first-order condition is

$$
\frac{q_i(t)}{Q(t)} * F_t' + \left( 1 - \frac{q_i(t)}{Q(t)} \right) * \frac{F_t'}{Q(t)} - C_i'(q_i(t)) - \mu_{i,t} * F_t' = 0.
$$

Rearranging terms and using $\mu_{i,t} = \frac{q_i(t)}{Q(t)} \mu_t$ yields

$$
\frac{F_t(Q_t) - Q_t C_i'}{q_i(t)} = \frac{F_t(Q_t)}{Q_t} - (1 - \mu_t) * F_t',
$$

which is the same as Condition (3) except for the factor $(1- \mu_t)$ on the right hand side and so collapses to (3) when $\mu_t = 0$. Again noting that the right hand side is the same for all $i$, we again have the common ratio property given in Equation (4).

As the cost function is still convex, we also have the co-monotone property, so all the results of the static case flow through to the dynamic case as well. Just as we did not need to

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3 Note that this approach does not require solving for the value function.
solve for the actual Nash equilibria in the static setting, here we do not need to solve for the Markov perfect equilibria. The first-order conditions at each "stage" of the dynamic game alone are sufficient to generate testable hypotheses.

5. The Average Cost Game

The above models of the Tragedy of the Commons are all examples of what is known as the "average return game," in which firms choose their inputs and obtain proportionate shares of the joint output. An alternative property rights institution is the "average cost game," in which firms (or individuals) choose their level of service and all pay proportionate shares of the joint cost. Examples include telephony, computer networks, and common facilities like clubs.4

Suppose there are \( I \) firms or individuals that share the same resource. Each firm's service from the resource at time \( t \) is denoted as \( y_{i,t} \), with total output being \( Y_t \). Each agent has an individual-specific concave value function for services, \( u_i(y_{i,t}) \). \( C_t(Y_t) \) is the total cost of the total service level and each firm's cost is proportional to its output level. Thus firm \( i \)'s problem is:

\[
\max_{y_{i,t}} \left\{ u_i(y_{i,t}) - \frac{y_{i,t}}{Y_t} * C_t(Y_t) \right\}
\]

(15)

The first-order condition is

\[
\frac{y_{i,t}}{Y_t} * C_t'(Y_t) - \frac{y_{i,t}}{Y_t^2} * C_t(Y_t) + \frac{C_t(Y_t)}{Y_t} = u_{i,t}'.
\]

(16)

Rearranging terms we get:

\[
\frac{C_t(Y_t) - Y_t u_{i,t}'}{y_{i,t}} = \frac{C_t(Y_t)}{Y_t} - C_t'(Y_t).
\]

(17)

This expression is analogous to equation (3) with \( F_t(Q_t) \) replaced by the total cost \( C_t(Y_t) \) and \( C_{i,t}' \) is replaced by individual utility \( u_{i,t}' \). From Equation (17) we obtain the "common ratio property" as before:

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4 See Moulin and Watts (1997) for a comparison of the average cost and average return games.
Notice that the common ratio is now negative given the convexity of the cost function. Likewise, with concave, \( u(\cdot) \), the co-monotone property is now \( y_{i,t} > y_{i,t'} \rightarrow u'_{i,t} \leq u'_{i,t'} \). A panel data set \( O = \left\{ C_t, (y_{i,t})_{i \in \{1 \ldots N\}} \right\}_{t \in \{1 \ldots T\}} \) is consistent with the average-cost game if there exist concave value functions \( \tilde{u}_i \) for each firm \( i \), and a convex cost function \( \tilde{C}_t \) for each observation \( t \) satisfying these properties.

**Conclusion**

We show how data from a common pool resource can be used to test, non-parametrically, for behavior consistent with the tragedy of the commons in a variety of settings, including static or dynamic resources, Nash or consistent-conjectures behavior, and average-return or average-cost commons games. In future work, we will extend these results to stochastic production functions (such as a fishery in which catch is uncertain) and/or measurement error in the data. Eventually, our goal is to apply these tests to real-world data from a common pool resource such as a fishery.
References


