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The Theory of Rationally Heterogeneous Expectations:
Evidence from Survey Data on Inflation Expectations.
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Method:
A data set from a monthly survey of approximately 500 households conducted by the Survey Research Center (SRC) at the University of Michigan, in which participants were asked questions regarding their expectations for inflation[1], is used (via MLE) to infer how people predict inflation and the dynamics involved between predictions. In the model each person chooses a predictor function on which to base expectations of inflation. The choice is based on the benefit gained from a particular function, and the cost associated with each function; generally it is assumed that cost increases as the sophistication of the model increases. For the purposes of the paper Branch presents a restricted predictor choice set that allows for: Naïve Expectations (π_{t-1}); Adaptive Expectations (AE_t); and a Vector Autoregression[2] predictor (VAR_t).

Naïve Expectations (NE) represent the simplest form of inflationary expectations, in that $t+1$ period's inflation forecast is set as the inflation rate from period *t*:

$$(1.1)\ \pi_{t+12}^{e,N} = \pi_t^m .$$ [3]

The **Adaptive Expectations** (AE) predictor represents a more sophisticated model that uses prior inflation rates as well as prior AE:

$$(1.2)\ AE_t = AE_{t-1} + \gamma(\pi_{t-1} - AE_{t-1}) = \gamma\pi_{t-1} + (1-\gamma)AE_{t-1} .$$

Gamma is calculated by simulating the above equation for different values of γ using monthly inflation rates over the period from 1947-1999 (1947.01-1999.04 precisely) where inflation in period *t*=0 (π_0) is set to the monthly inflation rate of 1947: $\pi_o^A = \pi_{1947.01}^m$. The actual γ used in the predictor model of the paper was found by minimizing the MSE of expected and past inflation when using an AE predictor function:

[1] More detailed information on the survey can be found on page 603.
[2] Due to the fact that arguments for rational expectations are highly dubious (in that it requires people to posses too much knowledge,) the vector auotoregression function is used as a bounded proxy for rational expectations.
[3] Annual inflation is defined as: $\pi_t = 100 \times \ln\left(\frac{CPI_t}{CPI_{t-12}}\right)$, because most people observe monthly inflation rates, a monthly analog, $\pi_t^m = 1200 \times \ln\left(\frac{CPI_t}{CPI_{t-1}}\right)$ is used as the input for the predictor function.

$$(1.3)\ \gamma = \min_{\gamma}[MSE] = \min_{\gamma}\left\{\frac{1}{T}\left[\sum_{t=0}^{T}(\pi_t^{e,A} - \pi_{t+12})^2\right]\right\},$$

where T is the total number of time periods in the sample and $\gamma \in [0,1]$. Branch calculated that $\gamma = 0.216$ is the value that minimizes the MSE. Substituting this into the predictor function he finds that expected inflation using AE is:

$$(1.4)\ \pi_t^{e,A} = 0.216\pi_{t-1}^{m} + 0.784\pi_{t-1}^{e,A}.$$

This last predictor uses weights of past inflation and past AE expectations to form present expectations. It is noted that if $\gamma = 1$ then AE is equal to the NE predictor, meaning that all the weight is placed on recent inflation.

The **Vector Autoregression** (VAR) represents the most sophisticated of the predictor models presented in this paper:

$$(1.5)\ y_t = \alpha + \Omega_1 y_{t-1} + \Omega_2 y_{t-2} + ... + \Omega_\rho y_{t-\rho} + u_t$$

where y_t is the ($n \times 1$) vector of "inflation and other relevant variable that produce the best forecast of inflation" and $u_t \overset{iid}{\sim} N(0,Z)$. For an approximation of bounded rational expectations, Branch uses: monthly inflation, monthly unemployment rate, the monthly growth rate of M1, and the 3-month T-Bill rate as the variables for the VAR model (Branch does not argue that these are the perfect variables for producing a proxy for rational expectations, rather he emphasizes that the included variables should produce a more accurate forecast than that of the AE and NE.) Basically the VAR model is used to forecast each period 12 months ahead and the sum of these values forms the prediction for the future annual forecast: $\hat{\pi}_{t+12} = \sum_{i=1}^{12} \hat{\pi}_{t+i}^{m}$.

With a chosen predictor function the observed expectation of inflation becomes:

$$(1.6)\ \pi_{i,t}^{e} = H_j(\pi_t) + v_{i,t},$$

where $H_j(\pi_t) \in \{\pi_{t-1}, AE_t, VAR_t\}$ is the set of predictor functions available. The stochastic term $v_{i,t} \overset{iid}{\sim} N(0,\sigma_v^2)$ can "be attributed to small trembles to the agents' belief formation."

After using a predictor, people judge the success of their choice based on the cost of using the predictor (perhaps payment to a professional forecaster) and the benefits gained. Benefits are measured in the form a low MSE:

$$(1.7)\,MSE_{j,t} = (1-\delta)MSE_{j,t-1} + \delta(\pi^e_{i,t} - \pi_t)^2 .$$

Costs for the VAR, AE, and NE predictors are given by C_V, C_A, C_N respectively. It is assumed that the more sophisticated the predictor function, the higher the cost i.e. $C_V \geq C_A \geq C_N \geq 0$. Combined with the benefits, the success/utility of a given predictor is:

$$(1.8)\,U_{j,t} = -(MSE_{j,t} + C_j)$$

A predictor function is chosen based on its success relative to the success of the other predictor functions. The probability of choosing predictor function *j* is given by:

$$(1.9)\,\Pr\left(j \middle| U_{j,t}\right) = n_{j,t} = \frac{\exp\{\beta[-(MSE_{j,t} + C_j)]\}}{\sum_{k\in\{N,A,V\}} \exp\{\beta[-MSE_{k,t} + C_k)]\}},$$

Where β (the 'intensity of choice' parameter) represents some friction that slows change from one predictor to another, or conversely how quickly a person will switch between predictors to gain benefit. A higher level of β ($\beta \in \mathbb{R}_+$) will lead to quicker moves between predictors.

The Applied Models and Results:

To more precisely fit the data, an alternative description to the basic MSE presented above[4] is formulated, that effectively allows for a different value (weighting) of δ for each period:

$$(1.10)\,MSE_{j,t} = \left[MSE^{\delta=1}_{j,t-1}, MSE^{\delta=0.9}_{j,t-3}\right] = \left[\left(\pi^m_{t-1} - \pi^j_{t-2}\right)^2, .1MSE^{\delta=0.9}_{j,t-3} + 0.9\left(\pi^m_{t-2} - \pi^j_{t-3}\right)^2\right].$$

This, according to Branch, effectively dampens the effects of monthly volatility. "Here $MSE^{\delta=1}_{j,t-1}$ is the forecast error model *j* produces based on the most recent monthly inflation observation. $MSE^{\delta=0.9}_{j,t-3}$ is a geometrically weighted average of all past forecast errors."

Betta becomes a vector, $\beta = [\beta_1, \beta_2]'$, to account for how recent and past changes in MSE affect the propensity of a person to change predictor functions.

Model 1: *Dynamic model with no costs*

Model assumes that $C_V = C_A = C_N = 0$ therefore $(1.11)\,U_{j,t} = -\beta_1 MSE^{\delta=1}_{j,t-1} - \beta_2 MSE^{\delta=0.9}_{j,t}$. The MLE estimates the predictor proportions, β_1 and β_2 while ignoring any costs

4 $MSE_{j,t} = (1-\delta)MSE_{j,t-1} + \delta(\pi^e_{i,t} - \pi_t)^2$

associated with a predictor function. The results estimate $\beta_1 = 0.4594$ and $\beta_2 = 0.0356$, meaning that people will change more readily with the most recent forecast errors than with the dampened past errors.[5]

Model 2: *Static predictor selection*

Model assumes that people are predisposed to use a particular predictor function and will not change; excludes the dynamic changing effects and simply looks at the costs. The utility function becomes: $U_{j,t} = -C_j$. The MLE results show a baseline proportion for each predictor function to be: VAR=48%, AE=44% and NE=7%. Paradoxically the estimates of costs show that $C_N > C_A > C_V$ which contradicts the earlier prediction that the more sophisticated model would be most expensive.

Model 3: *Full Model*

Model combines the dynamic model (1) and the static model (2). The cost structure follows the same form as model 2, $C_N > C_A > C_V$ and Branch argues that these costs represent the predisposition to a particular predictor. Essentially Branch says that people are predisposed to a certain predictor (mostly to VAR and AE) but if the MSE of their preferred predictor increases past a threshold level, the gains from switching to a more accurate predictor outweigh the costs of doing so. The figures that depict the predictor portions[6] show variations around the proportions from model 2 (explained by an underlying predisposition toward a particular predictor function.) Simply, people are predisposed to a certain predictor, given an unacceptable increase in MSE of their predictor a person will switch predictors but tend strongly toward their preferred predictor.

Comments:

Branch's paper works as an extension to previous Adaptively Rational Equilibrium Dynamics (ARED) models. There are some discrepancies with previous assumptions (i.e. costs should increase with a predictor's sophistication) but the interpretation of the results explain why different predictor functions may be rational (optimal) for different people, pointing toward a plausible case for rationally heterogeneous expectations. With this in mind, it should be noted that when the full dynamic model is compared to the static model, the full model (with dynamic switching) explains the data more thoroughly.

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[5] The figures depicting predictor proportions for model 2 can be found on pages 608-609.
[6] The figures depicting predictor proportions for model 3 can be found on pages 614-615.