Valuation of Bonds and Stock

First Principles:
- Value of financial securities = PV of expected future cash flows
- To value bonds and stocks we need to:
  - Estimate future cash flows:
    - Size (how much) and
    - Timing (when)
  - Discount future cash flows at an appropriate rate:
    - The rate should be appropriate to the risk presented by the security.

Pure Discount Bonds: Example

Find the value of a 30-year zero-coupon bond with a $1,000 par value and a YTM of 6%.

Level-Coupon Bonds: Example

Find the present value (as of January 1, 2004), of a 6-3/8 coupon T-bond with semi-annual payments, and a maturity date of December 2009 if the YTM is 5-percent.
5.3 Bond Concepts

1. Bond prices and market interest rates move in opposite directions.
2. When coupon rate = YTM, price = par value. When coupon rate > YTM, price > par value (premium bond) When coupon rate < YTM, price < par value (discount bond)
3. A bond with longer maturity has higher relative (%) price change than one with shorter maturity when interest rate (YTM) changes. All other features are identical.
4. A lower coupon bond has a higher relative price change than a higher coupon bond when YTM changes. All other features are identical.

YTM and Bond Value

When the YTM < coupon, the bond trades at a premium. When the YTM = coupon, the bond trades at par. When the YTM > coupon, the bond trades at a discount.

Maturity and Bond Price Volatility

Consider two otherwise identical bonds. The long-maturity bond will have much more volatility with respect to changes in the discount rate.

Coupon Rate and Bond Price Volatility

Consider two otherwise identical bonds. The low-coupon bond will have much more volatility with respect to changes in the discount rate.

5.4 The Present Value of Common Stocks

- Dividends versus Capital Gains
- Valuation of Different Types of Stocks
  - Zero Growth
  - Constant Growth
  - Differential Growth

Case 1: Zero Growth

Assume that dividends will remain at the same level forever.

\[
Div_0 = Div_1 = Div_2 = \cdots
\]

Since future cash flows are constant, the value of a zero growth stock is the present value of a perpetuity:

\[
P_0 = \frac{Div_1}{1 + r} + \frac{Div_2}{(1 + r)^2} + \frac{Div_3}{(1 + r)^3} + \cdots
\]

\[
P_0 = \frac{Div}{r}
\]
Case 2: Constant Growth

Assume that dividends will grow at a constant rate, \( g \), forever. i.e.

\[
\begin{align*}
\text{Div}_1 &= \text{Div}_0 (1 + g) \\
\text{Div}_2 &= \text{Div}_0 (1 + g)^2 \\
\text{Div}_3 &= \text{Div}_0 (1 + g)^3 \\
\vdots \\
\text{Div}_n &= \text{Div}_0 (1 + g)^n
\end{align*}
\]

Since future cash flows grow at a constant rate forever, the value of a constant growth stock is the present value of a growing perpetuity:

\[
P_n = \frac{\text{Div}}{r - g}
\]

Case 3: Differential Growth

Assume that dividends will grow at different rates in the foreseeable future and then will grow at a constant rate thereafter.

To value a Differential Growth Stock, we need to:
- Estimate future dividends in the foreseeable future.
- Estimate the future stock price when the stock becomes a Constant Growth Stock (case 2).
- Compute the total present value of the estimated future dividends and future stock price at the appropriate discount rate.

A Differential Growth Example

A common stock just paid a dividend of $2. The dividend is expected to grow at 8% for 3 years, then it will grow at 4% in perpetuity.

What is the stock worth? The discount rate is 12%.

NPV of expected Dividends and Selling Price

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<td>2.52+32.75+35.27</td>
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<tr>
<td>3</td>
<td>2.32+32.75+35.27</td>
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</table>

NPV = $28.89

A Differential Growth Example (continued)

\[
P_0 = \frac{2.16}{0.12} + \frac{2.33}{(1.12)^2} + \frac{2.52 + 32.75}{(1.12)^3} = 28.89
\]

A Differential Growth Example (continued)

\[
N = 3, \quad I/Y = 3.70, \quad PV = 5.58, \quad PMT = 2, \quad FV = 32.75
\]

First find the PV of the supernormal dividend stream then find the PV of the steady-state dividend stream.

\[
P_0 = \frac{2}{1.08} + \frac{2}{(1.08)^2} + \frac{0}{(1.08)^3} = 28.89
\]
5.8 Price Earnings Ratio

- Many analysts frequently relate earnings per share to price.
- The price earnings ratio is a.k.a. the multiple
- Calculated as current stock price divided by annual EPS
- The Wall Street Journal uses last 4 quarter’s earnings

\[ P/E \text{ ratio} = \frac{\text{Price per share}}{\text{EPS}} \]

- Firms whose shares are “in fashion” sell at high multiples. Growth stocks for example.
- Firms whose shares are out of favor sell at low multiples. Value stocks for example.

Other Price Ratio Analysis

- Many analysts frequently relate earnings per share to variables other than price, e.g.:
  - Price/Cash Flow Ratio
    \[ \text{cash flow} = \text{net income} + \text{depreciation} = \text{cash flow from operations or operating cash flow} \]
  - Price/Sales
    \[ \text{current stock price} \div \text{annual sales per share} \]
  - Price/Book (a.k.a. Market to Book Ratio)
    \[ \text{price divided by book value of equity, which is measured as assets – liabilities} \]

5.10 Summary and Conclusions

To Value Bonds:
1. The value of a zero-coupon bond (PV=Price or Value, FV, I, PMT=0, N=number of years).
2. The value of a perpetuity (PV=Annual Coupon/I)
3. The value of a coupon bond is the sum of the PV of the annuity of coupon payments plus the PV of the par value at maturity.
   - Usually U.S. semi-annual; (PV, FV=1000, N=number of coupons remaining or years remaining times two, I=YTM/2, PMT= Annual coupon/2)

A stock can be valued by discounting its dividends. There are three cases:

- Zero growth in dividends \[ P_0 = \frac{\text{Div}}{r} \]
- Constant growth in dividends \[ P_0 = \frac{\text{Div}_1}{r - g} \]
- Differential growth in dividends, which are solved as uneven cash flow problems