Chapter 4
Discounted Cash Flow Valuation

http://www2.gsu.edu/~fnccwh/pdf/ch4jaffev3overview.pdf

http://www.westga.edu/~chodges/video/tvmhints/

Key Concepts and Skills

- Be able to compute the future value and/or present value of a single cash flow or series of cash flows
- Be able to compute the return on an investment
- Be able to use a financial calculator or spreadsheet to solve time value problems
- Understand perpetuities and annuities
- Be prepared for multi-step problems

Chapter Outline

4.1 Valuation: The One-Period Case
4.2 The Multiperiod Case
4.3 Compounding Periods
4.4 Simplifications
4.5 Loan Amortization
4.6 What Is a Firm Worth?

Problem Types

- Perpetuity/Growing Perpetuity
  - \( PV_n = \frac{(\text{Cash Flow}_n)}{(\text{Interest Rate} - \text{growth rate})} \)
- Time Value of Money (PV, FV, N, PMT, I)
- Uneven Cash Flows (NPV and IRR)
- Multi-Step Problems
  - Time Value of Money (solve for missing inputs before solving for answer)
  - Future Value of Uneven Cash Flows (solve for NPV of known cash flows, use this known cash flow to solve for unknown cash flows)

Time lines show timing of cash flows.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>N=3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PV=CF₀</td>
<td>PMT=CF₁</td>
<td>PMT=CF₂</td>
<td>PMT &amp; FV=CF₃</td>
</tr>
</tbody>
</table>

Tick marks at ends of periods, so Time 0 is today; Time 1 is the end of Period 1; or the beginning of Period 2.

Three Ways to Solve Problems

- Solve the equation with a regular calculator.
- Use a financial calculator.
- Use a spreadsheet.
- The Present Value of 75.13 will have a Future Value of 100.00 if received in three years at a 10% interest rate
Solve $FV_N = PV(1 + I)^N$ for $PV$

$PV = \frac{FV_N}{(1+I)^N} = FV_N \left( \frac{1}{1+I} \right)^N$

$PV = $100 \left( \frac{1}{1.10} \right)^3 = $100(0.7513) = $75.13

Financial Calculator Solution

Either $PV$ or $FV$ must be negative. Here $PV = -$75.13. Put in $75.13$ today, take out $100$ after 3 years.

http://www.tvmcalcs.com/calculator_index

Continuous Compounding

The general formula for the future value of an investment compounded continuously over many periods can be written as:

$FV = C_0 e^{rT}$

Where $C_0$ is cash flow at date 0, $r$ is the stated annual interest rate, $T$ is the number of years, and $e$ is a transcendental number approximately equal to 2.718. $e^r$ is a key on your calculator.

Perpetuity

A constant stream of cash flows that lasts forever

Growing perpetuity

A stream of cash flows that grows at a constant rate forever

Annuity

A stream of constant cash flows that lasts for a fixed number of periods

Growing annuity

A stream of cash flows that grows at a constant rate for a fixed number of periods

Perpetuity

$PV = \frac{C}{r}$
Growing Perpetuity

A growing stream of cash flows that lasts forever

\[ C \times (1+g) \times (1+g)^2 \times (1+g)^3 \times \ldots \]

\[ PV = \frac{C}{r-g} \]

Growing Annuity

A growing stream of cash flows with a fixed maturity

\[ C \times (1+g) \times (1+g)^2 \times (1+g)^3 \times \ldots \]

\[ PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \frac{C \times (1+g)^2}{(1+r)^3} + \ldots \]

Growing Annuity: Example

A defined-benefit retirement plan offers to pay $20,000 per year for 40 years and increase the annual payment by 3% each year. What is the present value at retirement if the discount rate is 10%?

\[ PV = \frac{20,000}{.10-.03} \left[ 1 - \left( \frac{1.03}{1.10} \right)^{40} \right] = $265,121.57 \]

Amortized Loan with Fixed Payment

- Each payment covers the interest expense plus reduces principal
- Key relationships
  - Beginning Balance = Previous Ending Balance
  - Payment = Principal + Interest
  - Interest = Beginning Balance \times Interest Rate
  - Beginning Balance – Principal = Ending Balance
- Consider a 4 year loan with annual payments. The interest rate is 8%, and the principal amount is $5,000. What is the annual payment?
  - 4 N, 8 I/Y, 5,000 PV, CPT PMT = -1,509.60

Amortized Loan

<table>
<thead>
<tr>
<th>Year</th>
<th>Beginning Balance</th>
<th>Total Payment</th>
<th>Interest Paid</th>
<th>Principal Paid</th>
<th>Ending Balance</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>5,000.00</td>
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<tr>
<td>Totals</td>
<td>6,038.40</td>
<td>1,038.41</td>
<td>4,999.99</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.5 Loan Amortization

- Pure Discount Loans are the simplest form of loan. The borrower receives money today and repays a single lump sum (principal and interest) at a future time.
- Interest-Only Loans require an interest payment each period, with full principal due at maturity.
- Amortized Loans require repayment of principal over time, in addition to required interest.
4.6 What Is a Firm Worth?

- Conceptually, a firm should be worth the present value of the firm’s cash flows.
- The tricky part is determining the size, timing, and risk of those cash flows.

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