Chapter 17
Option Pricing with Applications to Real Options
ANSWERS TO SELECTED END-OF-CHAPTER QUESTIONS

17-1 a. An option is a contract which gives its holder the right to buy or sell an asset at some predetermined price within a specified period of time. A call option allows the holder to buy the asset, while a put option allows the holder to sell the asset.

b. A simple measure of an option’s value is its exercise value. The exercise value is equal to the current price of the stock (underlying the option) less the striking price of the option. The strike price is the price stated in the option contract at which the security can be bought (or sold). For example, if the underlying stock sells for $50 and the striking price is $20, the exercise value of the option would be $30.

c. The Black-Scholes Option Pricing Model is widely used by option traders to value options. It is derived from the concept of a riskless hedge. By buying shares of a stock and simultaneously selling call options on that stock, the investor will create a risk-free investment position. This riskless return must equal the risk-free rate or an arbitrage opportunity would exist. People would take advantage of this opportunity until the equilibrium level estimated by the Black-Scholes model was reached.

d. Real options occur when managers can influence the size and risk of a project’s cash flows by taking different actions during the project’s life. They are referred to as real options because they deal with real as opposed to financial assets. They are also called managerial options because they give opportunities to managers to respond to changing market conditions. Sometimes they are called strategic options because they often deal with strategic issues. Finally, they are also called embedded options because they are a part of another project.

e. Investment timing options give companies the option to delay a project rather than implement it immediately. This option to wait allows a company to reduce the uncertainty of market conditions before it decides to implement the project. Capacity options allow a company to change the capacity of their output in response to changing market conditions. This includes the option to contract or expand production. It also includes the option to abandon a project if market conditions deteriorate too much.

f. Decision trees are a form of scenario analysis in which different actions are taken in different scenarios.
g. Growth options allow a company to expand if market demand is higher than expected. This includes the opportunity to expand into different geographic markets and the opportunity to introduce complementary or second-generation products.

17-2 The market value of an option is typically higher than its exercise value due to the speculative nature of the investment. Options allow investors to gain a high degree of personal leverage when buying securities. The option allows the investor to limit his or her loss but amplify his or her return. The exact amount this protection is worth is the premium over the exercise value.

17-3 Postponing the project means that cash flows come later rather than sooner; however, waiting may allow you to take advantage of changing conditions. It might make sense, however, to proceed today if there are important advantages to being the first competitor to enter a market.

17-4 Timing options make it less likely that a project will be accepted today. Often, if a firm can delay a decision, it can increase the expected NPV of a project.

17-5 Having the option to abandon a project makes it more likely that the project will be accepted today.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

17-1 \( P = \$15; X = \$15; t = 0.5; r_{RF} = 0.10; \sigma^2 = 0.12; d_1 = 0.32660; d_2 = 0.08165; N(d_1) = 0.62795; N(d_2) = 0.53252; V = ? \)

Using the Black-Scholes Option Pricing Model, you calculate the option’s value as:

\[
V = P[N(d_1)] - Xe^{-r_{RF}t}[N(d_2)]
\]

\[
= \$15(0.62795) - \$15e^{-0.10(0.5)}(0.53252)
\]

\[
= \$9.4193 - \$15(0.9512)(0.53252)
\]

\[
= \$1.8211 \approx \$1.82.
\]

17-2 Option’s exercise price = \$15; Exercise value = \$22; Premium value = \$5;

\( V = ? \) \( P_0 = ? \)

\[
\text{Premium} = \text{Market price of option} - \text{Exercise value}
\]

\[
\$5 = V - \$22
\]

\[
V = \$27.
\]

Exercise value = \( P_0 - \text{Exercise price} \)

Mini Case: 17 - 2
$22 = P_0 - $15

P_0 = $37.
17-3 a. 

\[ \begin{array}{cccc} 
0 & 1 & 2 & 20 \\
-20 & 3 & 3 & 3 \\
\end{array} \]

NPV = $1.074 million.

b. Wait 1 year:

\[
\begin{array}{ccccccc}
0 & r=13% & 1 & 2 & 3 & 21 & \\
Tax imposed & 0 & -20 & 2.2 & 2.2 & 2.2 & 15.45 \\
50% Prob. & & & & & & \\
Tax not imposed & 0 & -20 & 3.8 & 3.8 & 3.8 & 26.69 \\
50% Prob. & & & & & & \\
\end{array}
\]

Tax imposed: NPV @ Yr. 1 = \((-20 + 15.45)/(1.13) = -4.027\)
Tax not imposed: NPV @ Yr 1 = \((-20 + 26.69)/(1.13) = 5.920\)
Expected NPV = \(0.5(-4.027) + 0.5(5.920) = 0.947\)

Note though, that if the tax is imposed, the NPV of the project is negative and therefore would not be undertaken. The value of this option of waiting one year is evaluated as \(0.5(0) + (0.5)(5.920) = 2.96\) million.

Since the NPV of waiting one year is greater than going ahead and proceeding with the project today, it makes sense to wait.

17-4 a. 

\[ \begin{array}{cccc} 
0 & 1 & 2 & 3 & 4 \\
-8 & 4 & 4 & 4 & 4 \\
\end{array} \]

NPV = $4.6795 million.

b. Wait 2 years:

\[
\begin{array}{cccccccc}
0 & r=10% & 1 & 2 & 3 & 4 & 5 & 6 \\
10% Prob. & 0 & 0 & -9 & 2.2 & 2.2 & 2.2 & 2.2 & 6.974 \\
90% Prob. & 0 & 0 & -9 & 4.2 & 4.2 & 4.2 & 4.2 & 13.313 \\
\end{array}
\]

Low CF scenario: NPV = \((-9 + 6.974)/(1.1)^2 = -1.674\)
High CF scenario: NPV = \((-9 + 13.313)/(1.1)^2 = 3.564\)
Expected NPV = \(0.1(-1.674) + 0.9(3.564) = 3.040\)

If the cash flows are only $2.2 million, the NPV of the project is negative and, thus, would not be undertaken. The value of the option of waiting two years is evaluated as \(0.10(0) + 0.90(3.564) = 3.208\) million.

*Mini Case: 17 - 4*
Since the NPV of waiting two years is less than going ahead and proceeding with the project today, it makes sense to drill today.

17-5  a.  0  1  2  20
     -300  40  40  40


b. Wait 1 year:

If the cash flows are only $30 million per year, the NPV of the project is negative. However, we’ve not considered the fact that the company could then be sold for $280 million. The decision tree would then look like this:

The expected NPV of waiting 1 year is 0.5(-$27.1468) + 0.5($45.3430) = $9.0981 million. Since the upper branch is negative, the optimal strategy is to wait and implement only if sales are $50 million per year. The NPV is: 0.5(0) + 0.5($45.3430) = $22.67 million.
Using a financial calculator, input the following data: CF₀ = -6,200,000; CF₁⁻¹⁵ = 600,000; I = 12; and then solve for NPV = -$2,113,481.31.

Using a financial calculator, input the following data: CF₀ = -6,200,000; CF₁⁻¹⁵ = 1,200,000; I = 12; and then solve for NPV = $1,973,037.39.

c. If they proceed with the project today, the project’s expected NPV = (0.5 × -$2,113,481.31) + (0.5 × $1,973,037.39) = -$70,221.96. So, Hart Enterprises would not do it.

d. Since the project’s NPV with the tax is negative, if the tax were imposed the firm would abandon the project. Thus, the decision tree looks like this:

Yes, the existence of the abandonment option changes the expected NPV of the project from negative to positive. Given this option the firm would take on the project because its expected NPV is $565,090.13.

e. If the firm pays $1,116,071.43 for the option to purchase the land, then the NPV of the project is exactly equal to zero. So the firm would not pay any more than this for the option.
17-7  \( P = \) PV of all expected future cash flows if project is delayed. From Problem 15-3 we know that PV @ Year 1 of Tax Imposed scenario is $15.45 and PV @ Year 1 of Tax Not Imposed Scenario is $26.69. So the PV is:

\[
P = \frac{[0.5(15.45)+ 0.5(26.69)]}{1.13} = $18.646.
\]

\( X = $20. \)

\( t = 1. \)

\( r_{RF} = 0.08. \)

\( \sigma^2 = 0.0687. \)

\[
d_1 = \ln\left[18.646/20\right] + [0.08 + .5(0.0687)](1) = 0.1688
\]

\[
d_2 = 0.1688 - (0.0687^{0.5} (1)^{0.5} = -0.0933
\]

From Excel function NORMSDIST, or approximated from Table 7E-1 in Extension to Chapter 7:

\( N(d_1) = 0.5670 \)

\( N(d_2) = 0.4628 \)

Using the Black-Scholes Option Pricing Model, you calculate the option’s value as:

\[
V = P[N(d_1)] - X e^{-rRFt} [N(d_2)]
\]

\[
= $18.646(0.5670) - $20e^{-0.08(1)}(0.4628)
\]

\[
= $10.572 - $8.544
\]

\[
= $2.028 million.
\]
17-8  \( P = \text{PV of all expected future cash flows if project is delayed.} \) From Problem 15-4 we know that PV @ Year 2 of Low CF Scenario is $6.974 and PV @ Year 2 of High CF Scenario is $13.313. So the PV is:

\[
P = \frac{0.1(6.974) + 0.9(13.313)}{1.10^2} = \$10.479.
\]

\( X = $9. \)

\( t = 2. \)

\( \sigma^2 = 0.0111. \)

\[
d_1 = \ln\left(\frac{10.479}{9}\right) + \left[0.06 + .5(.0111)\right](2) = 1.9010
\]

\[
d_2 = 1.9010 - (.0111)^{0.5}(2)^{0.5} = 1.7520
\]

From Excel function NORMSDIST, or approximated from Table 7E-1 in Extension to Chapter 7:

\( N(d_1) = 0.9713 \)

\( N(d_2) = 0.9601 \)

Using the Black-Scholes Option Pricing Model, you calculate the option’s value as:

\[
V = P[N(d_1)] - Xe^{-rt} [N(d_2)]
\]

\[
= 10.479(0.9713) - 9e^{-0.06}(2)(0.9601)
\]

\[
= 10.178 - 7.664
\]

\[
= $2.514 \text{ million.}
\]