Chapter 4
Discounted Cash Flow Valuation

http://www2.gsu.edu/~fnccwh/pdf/ch4jaffev3overview.pdf
http://www.westga.edu/~chodges/video/tvmhints/

Key Concepts and Skills

- Be able to compute the future value and/or present value of a single cash flow or series of cash flows
- Be able to compute the return on an investment
- Be able to use a financial calculator or spreadsheet to solve time value problems
- Understand perpetuities and annuities
- Be prepared for multi-step problems

Problem Types

- Perpetuity/Growing Perpetuity
  - \( PV_n = \frac{(Cash\ Flow_n)}{(Interest\ Rate - growth\ rate)} \)
- Time Value of Money (PV, FV, N, PMT, I)
- Uneven Cash Flows (NPV and IRR)
- Multi-Step Problems
  - Time Value of Money (solve for missing inputs before solving for answer)
  - Future Value of Uneven Cash Flows (solve for NPV of known cash flows, use this known cash flow to solve for unknown cash flows)

Time lines show timing of cash flows.

\[
\begin{array}{cccc}
0 & 1 & 2 & N=3 \\
| & | & | \\
\text{1\%} & PMT=CF_1 & PMT=CF_2 & PMT& CF_3 \\
\text{PV}=CF_0 & FV=CF_3 \\
\end{array}
\]

- Solve the equation with a regular calculator.
- Use a financial calculator.
- Use a spreadsheet.
- The Present Value of 75.13 will have a Future Value of 100.00 if received in three years at a 10% interest rate
Solve $FV_N = PV(1 + I)^N$ for $PV$

$$PV = \frac{FV_N}{(1+I)^N} = FV_N \left( \frac{1}{1 + I} \right)^N$$

$$PV = \frac{100}{1.10^3} = \frac{100}{1.331} = 75.13$$

Financial Calculator Solution

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>3</th>
<th>10</th>
<th>0</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>N</td>
<td>I/YR</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>OUTPUT</td>
<td>-75.13</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Either PV or FV must be negative. Here PV = -75.13. Put in $75.13$ today, take out $100$ after 3 years.
http://www.tvmcalcs.com/calculator_index

Spreadsheet Solution

- $= FV(I, N, PMT, PV, type)$
- $= PV(0.10, 3, 0, -100.00) = 75.13$

http://spreadsheets.about.com/od/excel101/Excel_101_How_to_Use_Excel_Spreadsheets.htm

Continuous Compounding

The general formula for the future value of an investment compounded continuously over many periods can be written as:

$$FV = C_0 e^{rT}$$

Where
- $C_0$ is cash flow at date 0,
- $r$ is the stated annual interest rate,
- $T$ is the number of years, and
- $e$ is a transcendental number approximately equal to 2.718. $e^r$ is a key on your calculator.

4.4 Simplifications

- Perpetuity
  - A constant stream of cash flows that lasts forever
- Growing perpetuity
  - A stream of cash flows that grows at a constant rate forever
- Annuity
  - A stream of constant cash flows that lasts for a fixed number of periods
- Growing annuity
  - A stream of cash flows that grows at a constant rate for a fixed number of periods

Perpetuity

A constant stream of cash flows that lasts forever

$$PV = \frac{C}{1} + \frac{C}{(1 + r)} + \frac{C}{(1 + r)^2} + \cdots$$

$$PV = \frac{C}{r}$$
Growing Perpetuity

A growing stream of cash flows that lasts forever

\[ \begin{align*}
C & \quad C \times (1 + g) \quad C \times (1 + g)^2 \quad \cdots \\
0 & \quad 1 & \quad 2 & \quad 3 & \quad \cdots
\end{align*} \]

\[ PV = \frac{C}{(1 + r)} + \frac{C \times (1 + g)}{(1 + r)^2} + \frac{C \times (1 + g)^2}{(1 + r)^3} + \cdots \]

\[ PV = \frac{C}{r - g} \]

Growing Annuity

A growing stream of cash flows with a fixed maturity

\[ \begin{align*}
C & \quad C \times (1 + g) \quad C \times (1 + g)^2 \quad \cdots \\
0 & \quad 1 & \quad 2 & \quad 3 & \quad \cdots & \quad T
\end{align*} \]

\[ PV = \frac{C}{(1 + r)} + \frac{C \times (1 + g)}{(1 + r)^2} + \cdots + \frac{C \times (1 + g)^{T-1}}{(1 + r)^T} \]

\[ PV = \frac{C}{r - g} \left[ 1 - \left( \frac{1 + g}{1 + r} \right)^T \right] \]

Growing Annuity: Example

A defined-benefit retirement plan offers to pay $20,000 per year for 40 years and increase the annual payment by 3% each year. What is the present value at retirement if the discount rate is 10%?

\[ \begin{align*}
0 & \quad 1 & \quad 2 & \quad \cdots & \quad 40 \\
\$20,000 & \quad \$20,000 \times (1.03) & \quad \$20,000 \times (1.03)^2 & \quad \cdots & \quad \$20,000 \times (1.03)^{39}
\end{align*} \]

\[ PV = \frac{\$20,000}{0.10 - 0.03} \left[ 1 - \left( \frac{1.03}{1.10} \right)^{40} \right] = \$265,121.57 \]

4.5 Loan Amortization

- Pure Discount Loans are the simplest form of loan. The borrower receives money today and repays a single lump sum (principal and interest) at a future time.
- Interest-Only Loans require an interest payment each period, with full principal due at maturity.
- Amortized Loans require repayment of principal over time, in addition to required interest.

Amortized Loan with Fixed Payment

- Each payment covers the interest expense plus reduces principal
- Key relationships
  - Beginning Balance = Previous Ending Balance
  - Payment = Principal + Interest
  - Interest = Beginning Balance \times Interest Rate
  - Beginning Balance = Principal + Ending Balance
- Consider a 4 year loan with annual payments. The interest rate is 8%, and the principal amount is $5,000. What is the annual payment?
  - 4 N, 8 I/Y, 5,000 PV, CPT PMT = -1,509.60

Amortized Loan

<table>
<thead>
<tr>
<th>Year</th>
<th>Beginning Balance</th>
<th>Total Payment</th>
<th>Interest Paid</th>
<th>Principal Paid</th>
<th>Ending Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5,000.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5,000.00</td>
</tr>
<tr>
<td>1</td>
<td>5,000.00</td>
<td>1,509.60</td>
<td>400.00</td>
<td>1,109.60</td>
<td>3,890.40</td>
</tr>
<tr>
<td>2</td>
<td>3,890.40</td>
<td>1,509.60</td>
<td>311.23</td>
<td>1,198.37</td>
<td>2,692.03</td>
</tr>
<tr>
<td>3</td>
<td>2,692.03</td>
<td>1,509.60</td>
<td>215.36</td>
<td>1,284.24</td>
<td>1,397.79</td>
</tr>
<tr>
<td>4</td>
<td>1,397.79</td>
<td>1,509.60</td>
<td>111.82</td>
<td>1,397.78</td>
<td>0.01</td>
</tr>
<tr>
<td>Totals</td>
<td>6,038.40</td>
<td>6,038.41</td>
<td>1,038.41</td>
<td>4,999.99</td>
<td></td>
</tr>
</tbody>
</table>
4.6 What Is a Firm Worth?

- Conceptually, a firm should be worth the present value of the firm's cash flows.
- The tricky part is determining the size, timing, and risk of those cash flows.

Contact Information

- Office: RCOB 18, U. of West Georgia
- Office Phone and Voicemail: (770)301-8648 (cell) or (678)839-4816 (office)
- Class Webpage: Ulearn
- E-mail: Ulearn (preferred) or chodges@westga.edu or chodges@gsu.edu
- Social Networking: Facebook, LinkedIn, and Instant Messenger
  mba8622@hotmail.com