Introduction to Valuation: The Time Value of Money

Key Concepts and Skills
- Be able to compute the future value of an investment made today
- Be able to compute the present value of cash to be received at some future date
- Be able to compute the return on an investment
- Be able to compute the number of periods that equates a present value and a future value given an interest rate
- Be able to use a financial calculator and a spreadsheet to solve time value of money problems

Chapter Outline
- Future Value and Compounding
- Present Value and Discounting
- More on Present and Future Values

Basic Definitions
- Present Value – earlier money on a time line
- Future Value – later money on a time line
- Interest rate – "exchange rate" between earlier money and later money
  - Discount rate
  - Cost of capital
  - Opportunity cost of capital
  - Required return

Table 5.4

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$FV = PV(1 + r)^n$</td>
<td>Future Value of $P$ invested at $r$ percent per period for $n$ periods.</td>
</tr>
<tr>
<td>$PV = FV / (1 + r)^n$</td>
<td>Present Value of $F$ to be received at $r$ percent per period.</td>
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Future Values
- Suppose you invest $1,000 for one year at 5% per year. What is the future value in one year?
  - Interest = $1,000(0.05) = 50$
  - Value in one year = principal + interest = $1,000 + 50 = 1,050$
  - Future Value (FV) = $1,000(1 + .05) = 1,050$
- Suppose you leave the money in for another year. How much will you have two years from now?
  - FV = $1,000(1.05)(1.05) = 1,000(1.05)^2 = 1,102.50$
Future Values: General Formula

- \( FV = PV(1 + r)^t \)
  - \( FV \) = future value
  - \( PV \) = present value
  - \( r \) = period interest rate, expressed as a decimal
  - \( t \) = number of periods
- Future value interest factor = \((1 + r)^t\)

Effects of Compounding

- Simple interest
- Compound interest
- Consider the previous example
  - \( FV \) with simple interest = \( 1,000 + 50 + 50 = 1,100 \)
  - \( FV \) with compound interest = \( 1,102.50 \)
  - The extra 2.50 comes from the interest of \(.05(50) = 2.50\) earned on the first interest payment

Calculator Keys

- Texas Instruments BA-II Plus
  - \( FV \) = future value
  - \( PV \) = present value
  - \( I/Y \) = period interest rate
  - \( P/Y \) must equal 1 for the \( I/Y \) to be the period rate
  - Interest is entered as number of percent, not a decimal
  - \( N \) = number of periods
  - Remember to clear the registers (CLR TVM) after each problem
  - Other calculators are similar in format

Future Values – Example 2

- Suppose you invest the $1,000 from the previous example for 5 years. How much would you have?
  - \( 5 \text{ N}; 5 \text{ I/Y}; 1,000 \text{ PV} \)
  - \( \text{CPT FV} = 1,276.28 \)
- The effect of compounding is small for a small number of periods, but increases as the number of periods increases. (Simple interest would have a future value of \$1,250, for a difference of $26.28.)

Future Values – Example 3

- Suppose you had a relative deposit $10 at 5.5% interest 200 years ago. How much would the investment be worth today?
  - \( 200 \text{ N}; 5.5 \text{ I/Y}; -10 \text{ PV} \)
  - \( \text{CPT FV} = 447,189.84 \)
- What is the effect of compounding?
  - Simple interest = \( 10 + 200(10)(.055) = 120.00 \)
  - Compounding added $447,069.84 to the value of the investment

Future Value as a General Growth Formula

- Suppose your company expects to increase unit sales of widgets by 15% per year for the next 5 years. If you sell 3 million widgets in the current year, how many widgets do you expect to sell in the fifth year?
  - \( 5 \text{ N}; 15 \text{ I/Y}; 3,000,000 \text{ PV} \)
  - \( \text{CPT FV} = -6,034,072 \text{ units (remember the sign convention)} \)
Quick Quiz – Part I

- What is the difference between simple interest and compound interest?
- Suppose you have $500 to invest and you believe that you can earn 8% per year over the next 15 years.
  - How much would you have at the end of 15 years using compound interest?
  - How much would you have using simple interest?

Present Values

- How much do I have to invest today to have some amount in the future?
  - $FV = PV(1 + r)^t$
  - Rearrange to solve for $PV = FV / (1 + r)^t$
- When we talk about discounting, we mean finding the present value of some future amount.
- When we talk about the “value” of something, we are talking about the present value unless we specifically indicate that we want the future value.

Present Value – One Period Example

- Suppose you need $10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?
- $PV = 10,000 / (1.07)^1 = 9,345.79$
- Calculator
  - 1 N; 7 I/Y; 10,000 FV
  - CPT PV = 9,345.79

Present Values – Example 2

- You want to begin saving for your daughter’s college education and you estimate that she will need $150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?
- $N = 17; I/Y = 8; FV = 150,000$
  - CPT PV = -40,540.34 (remember the sign convention)

Present Values – Example 3

- Your parents set up a trust fund for you 10 years ago that is now worth $19,671.51. If the fund earned 7% per year, how much did your parents invest?
  - $N = 10; I/Y = 7; FV = 19,671.51$
  - CPT PV = -10,000

Present Value – Important Relationship I

- For a given interest rate – the longer the time period, the lower the present value
  - What is the present value of $500 to be received in 5 years? 10 years? The discount rate is 10%
    - 5 years: $N = 5; I/Y = 10; FV = 500$
      - CPT PV = -310.46
    - 10 years: $N = 10; I/Y = 10; FV = 500$
      - CPT PV = -192.77
Present Value – Important Relationship II

- For a given time period – the higher the interest rate, the smaller the present value.
  - What is the present value of $500 received in 5 years if the interest rate is 10%? 15%?
    - Rate = 10%; N = 5; I/Y = 10; FV = 500
      CPT PV = -310.46
    - Rate = 15%; N = 5; I/Y = 15; FV = 500
      CPT PV = -248.59

Quick Quiz – Part II

- What is the relationship between present value and future value?
- Suppose you need $15,000 in 3 years. If you can earn 6% annually, how much do you need to invest today?
- If you could invest the money at 8%, would you have to invest more or less than at 6%? How much?

The Basic PV Equation - Refresher

- PV = FV / (1 + r)^t
- There are four parts to this equation:
  - PV, FV, r and t
  - If we know any three, we can solve for the fourth.
- If you are using a financial calculator, be sure and remember the sign convention or you will receive an error (or a nonsense answer) when solving for r or t.

Discount Rate

- Often we will want to know what the implied interest rate is on an investment.
- Rearrange the basic PV equation and solve for r:
  - FV = PV(1 + r)^t
  - r = (FV / PV)^1/t - 1
- If you are using formulas, you will want to make use of both the y^x and the 1/x keys.

Discount Rate – Example 1

- You are looking at an investment that will pay $1,200 in 5 years if you invest $1,000 today. What is the implied rate of interest?
  - r = (1,200 / 1,000)^1/5 - 1 = .03714 = 3.714%
  - Calculator – the sign convention matters!!!
    - N = 5
    - PV = -1,000 (you pay 1,000 today)
    - FV = 1,200 (you receive 1,200 in 5 years)
    - CPT I/Y = 3.714%

Discount Rate – Example 2

- Suppose you are offered an investment that will allow you to double your money in 6 years. You have $10,000 to invest. What is the implied rate of interest?
  - N = 6
  - PV = -10,000
  - FV = 20,000
  - CPT I/Y = 12.25%
Discount Rate – Example 3

- Suppose you have a 1-year old son and you want to provide $75,000 in 17 years towards his college education. You currently have $5,000 to invest. What interest rate must you earn to have the $75,000 when you need it?
  - \( N = 17; PV = -5,000; FV = 75,000 \)
  - CPT I/Y = 17.27%

Quick Quiz – Part III

- What are some situations in which you might want to know the implied interest rate?
- You are offered the following investments:
  - You can invest $500 today and receive $600 in 5 years. The investment is low risk.
  - You can invest the $500 in a bank account paying 4%.
  - What is the implied interest rate for the first choice, and which investment should you choose?

Finding the Number of Periods

- Start with basic equation and solve for \( t \) (remember your logs)
  - \( FV = PV(1 + r)^t \)
  - \( t = \ln(FV / PV) / \ln(1 + r) \)
- You can use the financial keys on the calculator as well; just remember the sign convention.

Number of Periods – Example 1

- You want to purchase a new car and you are willing to pay $20,000. If you can invest at 10% per year and you currently have $15,000, how long will it be before you have enough money to pay cash for the car?
  - I/Y = 10; PV = -15,000; FV = 20,000
  - CPT N = 3.02 years

Number of Periods – Example 2

- Suppose you want to buy a new house. You currently have $15,000 and you figure you need to have a 10% down payment plus an additional 5% of the loan amount for closing costs. Assume the type of house you want will cost about $150,000 and you can earn 7.5% per year. How long will it be before you have enough money for the down payment and closing costs?
  - Down payment = 0.1(150,000) = 15,000
  - Closing costs = 0.05(150,000 − 15,000) = 6,750
  - Total needed = 15,000 + 6,750 = 21,750
  - Compute the number of periods
    - \( PV = -15,000; FV = 21,750; I/Y = 7.5 \)
    - CPT N = 5.14 years
  - Using the formula
    - \( t = \ln(21,750 / 15,000) / \ln(1.075) = 5.14 \text{ years} \)
Quick Quiz – Part IV

- When might you want to compute the number of periods?
- Suppose you want to buy some new furniture for your family room. You currently have $500 and the furniture you want costs $600. If you can earn 6%, how long will you have to wait if you don’t add any additional money?

Spreadsheet Example

- Use the following formulas for TVM calculations
  - FV(rate,nper,pmt,pv)
  - PV(rate,nper,pmt,fv)
  - RATE(nper,pmt,pv,fv)
  - NPER(rate,pmt,pv,fv)
- The formula icon is very useful when you can’t remember the exact formula
- Click on the Excel icon to open a spreadsheet containing four different examples.

Work the Web Example

- Many financial calculators are available online
- Click on the web surfer to go to Investopedia’s web site and work the following example:
  - You need $50,000 in 10 years. If you can earn 6% interest, how much do you need to invest today?
  - You should get $27,919.74

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