CHAPTER 5
INTRODUCTION TO VALUATION: THE TIME VALUE OF MONEY

Answers to Concepts Review and Critical Thinking Questions

1. The four parts are the present value (PV), the future value (FV), the discount rate \( r \), and the life of the investment \( t \).

2. Compounding refers to the growth of a dollar amount through time via reinvestment of interest earned. It is also the process of determining the future value of an investment. Discounting is the process of determining the value today of an amount to be received in the future.

3. Future values grow (assuming a positive rate of return); present values shrink.

4. The future value rises (assuming it’s positive); the present value falls.

5. It would appear to be both deceptive and unethical to run such an ad without a disclaimer or explanation.

6. It’s a reflection of the time value of money. TMCC gets to use the $1,163. If TMCC uses it wisely, it will be worth more than $10,000 in thirty years.

7. This will probably make the security less desirable. TMCC will only repurchase the security prior to maturity if it is to its advantage, i.e. interest rates decline. Given the drop in interest rates needed to make this viable for TMCC, it is unlikely the company will repurchase the security. This is an example of a “call” feature. Such features are discussed at length in a later chapter.

8. The key considerations would be: (1) Is the rate of return implicit in the offer attractive relative to other, similar risk investments? and (2) How risky is the investment; i.e., how certain are we that we will actually get the $10,000? Thus, our answer does depend on who is making the promise to repay.

9. The Treasury security would have a somewhat higher price because the Treasury is the strongest of all borrowers.

10. The price would be higher because, as time passes, the price of the security will tend to rise toward $10,000. This rise is just a reflection of the time value of money. As time passes, the time until receipt of the $10,000 grows shorter, and the present value rises. In 2015, the price will probably be higher for the same reason. We cannot be sure, however, because interest rates could be much higher, or TMCC’s financial position could deteriorate. Either event would tend to depress the security’s price.
Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The simple interest per year is:

\[ \text{Interest} = \text{Principal} \times \text{Rate} \]

\[ \text{Interest} = 5,000 \times 0.06 = 300 \]

So after 10 years you will have:

\[ \text{Interest} = 300 \times 10 = 3,000 \]

The total balance will be $5,000 + 3,000 = 8,000

With compound interest we use the future value formula:

\[ FV = PV(1 + r)^t \]

\[ FV = 5,000(1.06)^10 = 8,954.24 \]

The difference is:

\[ 8,954.24 – 8,000 = 954.24 \]

2. To find the FV of a lump sum, we use:

\[ FV = PV(1 + r)^t \]

\[ FV = 2,250(1.10)^{16} = 10,338.69 \]

\[ FV = 8,752(1.08)^{13} = 23,802.15 \]

\[ FV = 76,355(1.17)^{4} = 143,080.66 \]

\[ FV = 183,796(1.07)^{12} = 413,943.81 \]

3. To find the PV of a lump sum, we use:

\[ PV = \frac{FV}{(1 + r)^t} \]

\[ PV = 15,451 / (1.04)^6 = 12,211.15 \]

\[ PV = 51,557 / (1.11)^7 = 24,832.86 \]

\[ PV = 886,073 / (1.20)^{23} = 13,375.22 \]

\[ PV = 550,164 / (1.13)^{18} = 60,964.94 \]
4. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[ FV = PV(1 + r)^t \]

Solving for \( r \), we get:

\[ r = (FV / PV)^{1/t} - 1 \]

\[ FV = $307 = $240(1 + r)^2; \quad r = ($307 / $240)^{1/2} - 1 = 13.10\% \]
\[ FV = $896 = $360(1 + r)^{10}; \quad r = ($896 / $360)^{1/10} - 1 = 9.55\% \]
\[ FV = $174,384 = $39,000(1 + r)^{15}; \quad r = ($174,384 / $39,000)^{1/15} - 1 = 10.50\% \]
\[ FV = $483,500 = $38,261(1 + r)^{30}; \quad r = ($483,500 / $38,261)^{1/30} - 1 = 8.82\% \]

5. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[ FV = PV(1 + r)^t \]

Solving for \( t \), we get:

\[ t = \ln(FV / PV) / \ln(1 + r) \]

\[ FV = $1,284 = $560(1.08)^t; \quad t = \ln($1,284 / $560) / \ln 1.08 = 10.78 \text{ years} \]
\[ FV = $4,341 = $810(1.09)^t; \quad t = \ln($4,341 / $810) / \ln 1.09 = 19.48 \text{ years} \]
\[ FV = $364,518 = $18,400(1.21)^t; \quad t = \ln($364,518 / $18,400) / \ln 1.21 = 15.67 \text{ years} \]
\[ FV = $173,439 = $21,500(1.13)^t; \quad t = \ln($173,439 / $21,500) / \ln 1.13 = 17.08 \text{ years} \]

6. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[ FV = PV(1 + r)^t \]

Solving for \( r \), we get:

\[ r = (FV / PV)^{1/t} - 1 \]
\[ r = ($280,000 / $50,000)^{1/18} - 1 = 10.04\% \]
7. To find the length of time for money to double, triple, etc., the present value and future value are irrelevant as long as the future value is twice the present value for doubling, three times as large for tripling, etc. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$$FV = PV(1 + r)^t$$

Solving for $t$, we get:

$$t = \frac{\ln(FV / PV)}{\ln(1 + r)}$$

The length of time to double your money is:

$$FV = $2 = $1(1.09)^t$$
$$t = \frac{\ln 2}{\ln 1.09} = 8.04 \text{ years}$$

The length of time to quadruple your money is:

$$FV = $4 = $1(1.09)^t$$
$$t = \frac{\ln 4}{\ln 1.09} = 16.09 \text{ years}$$

Notice that the length of time to quadruple your money is twice as long as the time needed to double your money (the difference in these answers is due to rounding). This is an important concept of time value of money.

8. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$$FV = PV(1 + r)^t$$

Solving for $r$, we get:

$$r = \left(\frac{FV}{PV}\right)^{1/t} - 1$$
$$r = \left(\frac{$27,958}{$21,608}\right)^{1/7} - 1 = 3.75\%$$

9. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$$FV = PV(1 + r)^t$$

Solving for $t$, we get:

$$t = \frac{\ln(FV / PV)}{\ln(1 + r)}$$
$$t = \frac{\ln ($170,000 / $40,000)}{\ln 1.062} = 24.05 \text{ years}$$

10. To find the PV of a lump sum, we use:

$$PV = \frac{FV}{(1 + r)^t}$$
$$PV = \frac{$700,000,000}{(1.085)^{20}} = $136,931,471.85$$
11. To find the PV of a lump sum, we use:

\[ PV = \frac{FV}{(1 + r)^t} \]

\[ PV = \frac{1,000,000}{(1.09)^{80}} = 1,013.63 \]

12. To find the FV of a lump sum, we use:

\[ FV = PV(1 + r)^t \]

\[ FV = 50(1.045)^{102} = 4,454.84 \]

13. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[ FV = PV(1 + r)^t \]

Solving for \( r \), we get:

\[ r = (\frac{FV}{PV})^{\frac{1}{t}} - 1 \]

\[ r = (\frac{1,170,000}{150})^{\frac{1}{111}} - 1 = 8.41\% \]

To find the FV of the first prize, we use:

\[ FV = PV(1 + r)^t \]

\[ FV = 1,170,000(1.0841)^{34} = 18,212,056.26 \]

14. To find the PV of a lump sum, we use:

\[ PV = \frac{FV}{(1 + r)^t} \]

\[ PV = \frac{485,000}{(1.2590)^{67}} = 0.10 \]

15. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[ FV = PV(1 + r)^t \]

Solving for \( r \), we get:

\[ r = (\frac{FV}{PV})^{\frac{1}{t}} - 1 \]

\[ r = (\frac{10,311,500}{12,377,500})^{\frac{1}{4}} - 1 = -4.46\% \]

Notice that the interest rate is negative. This occurs when the FV is less than the PV.
16. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[ FV = PV(1 + r)^t \]

Solving for \( r \), we get:

\[ r = (FV / PV)^{1/t} - 1 \]

a. \( PV = $10,000 / (1 + r)^{30} = $500 \)
\[ r = ($10,000 / $1,163)^{1/30} - 1 = 7.44\% \]

b. \( PV = $2,500 / (1 + r)^9 = $1,163 \)
\[ r = ($2,500 / $1,163)^{1/9} - 1 = 8.88\% \]

c. \( PV = $10,000 / (1 + r)^{21} = $2,500 \)
\[ r = ($10,000 / $2,500)^{1/21} - 1 = 6.82\% \]

17. To find the PV of a lump sum, we use:

\[ PV = FV / (1 + r)^t \]

\[ PV = $170,000 / (1.11)^{10} = $59,871.36 \]

18. To find the FV of a lump sum, we use:

\[ FV = PV(1 + r)^t \]

\[ FV = $2,000 (1.12)^{45} = $327,975.21 \]

\[ FV = $2,000 (1.12)^{35} = $105,599.24 \]

Better start early!

19. We need to find the FV of a lump sum. However, the money will only be invested for six years, so the number of periods is six.

\[ FV = PV(1 + r)^t \]

\[ FV = $25,000(1.079)^6 = $35,451.97 \]
To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[ FV = PV(1 + r)^t \]

Solving for \( t \), we get:

\[
t = \frac{\ln(FV / PV)}{\ln(1 + r)}
\]

\[
t = \frac{\ln(\$100,000 / \$10,000)}{\ln(1.11)} = 22.06
\]

So, the money must be invested for 22.06 years. However, you will not receive the money for another two years. From now, you’ll wait:

\[ 2 \text{ years} + 22.06 \text{ years} = 24.06 \text{ years} \]

**Calculator Solutions**

1. Enter \[ \begin{array}{cccc}
10 & \text{6}\% & \$5,000 \\
N & I/Y & PV & PMT & FV
\end{array} \]  
Solve for \$8,954.24

\$8,954.24 – 8,000 = \$954.24

2. Enter \[ \begin{array}{cccc}
16 & \text{10}\% & \$2,250 \\
N & I/Y & PV & PMT & FV
\end{array} \]  
Solve for \$10,338.69

3. Enter \[ \begin{array}{cccc}
6 & \text{4}\% & \\
N & I/Y & PV & PMT & FV
\end{array} \]  
Solve for \$12,211.15
Enter 7 11%  $51,557
Solve for $24,832.86

Enter 23 20%  $886,073
Solve for $13,375.22

Enter 18 13%  $550,164
Solve for $60,964.94

Enter 10 360 ±$307
Solve for 13.10%

Enter 15 39,000 ±$174,384
Solve for 10.50%

Enter 30 38,261 ±$483,500
Solve for 8.82%

Enter 8% $560 ±$1,284
Solve for 10.78

Enter 9% $810 ±$4,341
Solve for 19.48

Enter 21% $18,400 ±$364,518
Solve for 15.67
<table>
<thead>
<tr>
<th>Problem</th>
<th>Enter</th>
<th>Percent</th>
<th>Amount</th>
<th>± Amount</th>
<th>Solve for</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>18</td>
<td>13%</td>
<td>$21,500</td>
<td>±$173,439</td>
<td>17.08</td>
</tr>
<tr>
<td>7.</td>
<td>9%</td>
<td>$1</td>
<td></td>
<td>±$2</td>
<td>8.04</td>
</tr>
<tr>
<td>8.</td>
<td>7</td>
<td>8%</td>
<td>$21,608</td>
<td>±$27,958</td>
<td>3.75%</td>
</tr>
<tr>
<td>9.</td>
<td>6.20%</td>
<td>$40,000</td>
<td></td>
<td>±$170,000</td>
<td>24.05</td>
</tr>
<tr>
<td>10.</td>
<td>20</td>
<td>8.5%</td>
<td>$700,000,000</td>
<td></td>
<td>$136,931,471.85</td>
</tr>
<tr>
<td>11.</td>
<td>80</td>
<td>9%</td>
<td></td>
<td>$1,000,000</td>
<td>$1,013.63</td>
</tr>
<tr>
<td>12.</td>
<td>102</td>
<td>4.50%</td>
<td>$50</td>
<td></td>
<td>$4,454.84</td>
</tr>
</tbody>
</table>
13. Enter  
   111  ±$150 $1,170,000  
   N   I/Y   PV   PMT   FV  
Solve for  
   8.41%  

14. Enter  
   34  8.41% $1,170,000  
   N   I/Y   PV   PMT   FV  
Solve for  
   $18,212,056.26  

15. Enter  
   67  25.90% $485,000  
   N   I/Y   PV   PMT   FV  
Solve for  
   $0.10  

16. a. Enter  
   30  ±$12,377,500 $10,311,500  
   N   I/Y   PV   PMT   FV  
Solve for  
   7.44%  

16. b. Enter  
   9  ±$1,163 $2,500  
   N   I/Y   PV   PMT   FV  
Solve for  
   8.88%  

16. c. Enter  
   21  ±$2,500 $10,000  
   N   I/Y   PV   PMT   FV  
Solve for  
   6.82%  

17. Enter  
   10  11% $170,000  
   N   I/Y   PV   PMT   FV  
Solve for  
   $59,871.36  

18. Enter  
   45  12% $2,000  
   N   I/Y   PV   PMT   FV  
Solve for  
   $327,975.21  

Enter  
   35  12% $2,000  
   N   I/Y   PV   PMT   FV  
Solve for  
   $105,599.24
19. Enter 6 7.90% $25,000

Solve for PV PMT FV

Solve for $39,451.97

20. Enter 11% ±$10,000 $100,000

Solve for N I/Y PV PMT FV

Solve for 22.06

From now, you’ll wait 2 + 22.06 = 24.06 years