Exercise. In the Intel example verify that the risk-neutral expected return on Intel stock is the riskless rate of return, i.e.,

$$S = \frac{pS^+ + (1-p)S^-}{1 + r_f},$$  

(9)

where $S$, $S^+$, $S^-$, $r_f$ and $p$ are as in pages 602-606 in (BM).
Two Valuation Heuristics

Valuation

Dimensions

Time $\rightarrow$ Pricing Time

Risk $\rightarrow$ Pricing Risk
Two Valuation Heuristics

1. The one familiar from DCF analysis:

Market Value

\[ = \text{Expected Cash Flows} \]
\[ = \text{Discounted at Rate of Return Adjusted for the Risk of the Cash Flows} \]

2. The less familiar one (prevalent in option pricing):

Market Value

\[ = \text{Certainty Equivalent Cash Flows Discounted at the Riskless Rate of Return} \]
Heuristic – serving to indicate or point out, stimulating interest as a means of furthering investigation.
Risk-Neutral Valuation

The Absence of Arbitrage Opportunities

⇒ (and ⇐)

There is a change of probabilities (to risk-neutral probabilities) such that the price of any traded asset is equal to the expected cash flows from the asset (calculated using the risk-neutral probabilities) discounted at the risk free rate of return.
Arbitrage –

getting something

(positive payout or return)

for nothing

(a zero or negative investment).
Examples

(1) Suppose there are only two states of the world, and the payoffs on the only two assets in the world, A and B are as follows:

<table>
<thead>
<tr>
<th>ASSET</th>
<th>STATE 1</th>
<th>STATE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$20</td>
<td>$60</td>
</tr>
<tr>
<td>B</td>
<td>$40</td>
<td>$80</td>
</tr>
</tbody>
</table>

Suppose also that the risk free rate of return is 5%.

(i) Is there an arbitrage opportunity if A and B are currently selling at $30 and $32, respectively? If there does exist an arbitrage opportunity, construct an arbitrage portfolio.

(ii) Is there an arbitrage opportunity if A and B are currently selling at $30 and $70, respectively? If there does exist an arbitrage opportunity, construct an arbitrage portfolio.
(iii) Is there an arbitrage opportunity if A and B are currently selling at $30 and $45, respectively? If there does exist an arbitrage opportunity, construct an arbitrage portfolio.

(iv) Suppose assets A and B are currently selling for $45.60 and $64.6476 (to four decimal places), respectively. Is there an arbitrage opportunity? If not, what are the risk neutral probabilities of state 1 and state 2?

(v) Suppose that there was also a third security C, with state 1 and 2 payoffs of $30 and $200 respectively. What would its price be, in each of (i), (ii), (iii), (iv)?

Ans. (i) According to our principle of risk neutral valuation, there is an arbitrage opportunity if and only if we cannot find risk neutral probabilities such that the security prices are given as the expected cash flow discounted at the risk free rate of return. Let \( \pi_1 \) denote the risk neutral probability of state one
and let \( \pi_2 \) denote the risk neutral probability of state two. The condition we want is that expressed by the two simultaneous equations

\[
30 = (20 \pi_1 + 60 \pi_2)/1.05 \\
32 = (40 \pi_1 + 80 \pi_2)/1.05.
\]

Taking the 1.05 on the other side, these equations become

\[
31.50 = 20 \pi_1 + 60 \pi_2 \\
33.60 = 40 \pi_1 + 80 \pi_2.
\]

Subtracting the first equation from the second gives the equation

\[
2.10 = 20\pi_1 + 20 \pi_2,
\]

implying that \( \pi_1 + \pi_2 = 2.1/20 = .105 \), and hence there are no risk neutral probabilities. In fact, solving these equations we obtain \( \pi_1 = - 0.63 \) and \( \pi_2 = 0.735 \). Thus there exists an arbitrage
opportunity and it should be possible to construct an arbitrage portfolio. As the state 1 pure security has a negative price, a portfolio that pays off only in state 1 will require no investment and will generate a positive payoff at the end of the period. To solve for this portfolio, let $w_A (w_B)$ be the number of units of asset A (B) in the portfolio. Then it must be the case that $w_A 60 + w_B 80 = 0$, which implies that $w_A = -4/3 w_B$. Thus, to form an arbitrage portfolio, go short four units of asset A and go long in 3 units of asset B, i.e., $w_A = -4$ and $w_B = 3$. The portfolio pays 40 units in state 1 since $w_A 20 + w_B 40 = 40$ and zero in state 2 since $w_A 60 + w_B 80 = -240 + 240 = 0$. At the given prices, this portfolio cost $30(-4) + 32(3) = -120 + 96 = -24$.

(ii) To check for arbitrage solve

$$30 = (20 \pi_1 + 60 \pi_2)/1.05$$

$$70 = (40 \pi_1 + 80 \pi_2)/1.05.$$
But this implies that \( \pi_1 = 2.3625 \) and \( \pi_2 = -0.2625 \). Thus there exists an arbitrage opportunity. To construct an arbitrage portfolio construct a portfolio that only pays of in state 2, i.e. choose \( w_A \) and \( w_B \) such that \( 20w_A + 40w_B = 0 \) and such that \( 60w_A + 80w_B = 1000 \). Solving gives \( w_A = 50 \) and \( w_B = -25 \). The cost of this portfolio is \( 30(50) + 70(-25) = -250 \).

(iii) To check for arbitrage solve

\[
30 = \frac{(20 \pi_1 + 60 \pi_2)}{1.05}
\]

\[
45 = \frac{(40 \pi_1 + 80 \pi_2)}{1.05}.
\]

This implies that \( \pi_1 = \pi_2 = .39375 \). Thus there exists an arbitrage opportunity since \( \pi_1 + \pi_2 = .7875 < 1 \). In this case it is not easy to see what to sell and what to buy. Notice, however, that if the above were the risk neutral probabilities, then \$1.00 for certain would cost today \( (.39375 + .39375)/1.05 = .7875/1.05 = .75 \), where as the risk free rate being 5% implies that such a certain dollar would cost \( 1/1.05 = .9524 \). Thus we
could trade in security A and B and create certain cash flows cheaper than the market price. A portfolio of \( w_A = -50 \) and \( w_B = 50 \) would give $1000 for certain in the future and would cost \( 30(-50) + 45(50) = 750 \) today. Yet you could sell $1000 for certain in the future at the price of \( .9524(1000) = 952.40 \) today. The difference is $202.40 in arbitrage profit.

Notice that in this last example, the price of the riskless cash flow was too high relative to the prices of the other assets. What would happen if the price of the riskless asset was lower (the risk free rate of return was higher)? Suppose the risk free return is \( 1/3 \) (33 and \( 1/3\)%). Then the price of the riskless $1.00 in the future would be \( 1/1.33333\ldots = .75 \). In this case the above arbitrage opportunity is eliminated. To see this the equations for the risk neutral probabilities become

\[
30 = (20 \pi_1 + 60 \pi_2) \cdot .75
\]

\[
45 = (40 \pi_1 + 80 \pi_2) \cdot .75.
\]
Solving gives \( \pi_1 = \pi_2 = 0.5 \), and according to the principle of risk neutral valuation, there are no arbitrage opportunities.

(iv) To check for arbitrage solve the following equations:

\[
45.60 = \frac{(20 \pi_1 + 60 \pi_2)}{1.05}
\]

\[
64.6476 = \frac{(40 \pi_1 + 80 \pi_2)}{1.05}.
\]

This gives \( \pi_1 = .303 \) and \( \pi_2 = .697 \). Thus there are no arbitrage opportunities.

(v) In the case of parts (i), (ii), and (iii) with the risk free return of 5%, as there exist arbitrage opportunities, security C cannot be priced. In (iii) with the risk free return equal to 1/3%, the price of security C would be its expected cash flow using the risk neutral probabilities discounted at the risk free return or

\[
[0.5(30) + 0.5(200)] \cdot 0.75 = [115] \cdot 0.75 = 86.25.
\]
In (iv), the price of C must be the expected cash flow from C, calculated using the risk neutral probabilities $\pi_1 = .303$ and $\pi_2 = .697$, discounted at the risk free rate of return, i.e.,

$$\text{Price of C} = \frac{[.303(30) + .697(200)]}{1.05} = $141.42.$$

If the price of C was larger or smaller than $141.42, then by the principle of risk neutral valuation, there would be an arbitrage opportunity.

(2) Given that there are three states of nature, $s_1$, $s_2$, and $s_3$, risk neutral probabilities $\pi(s_1) = 0.3$, $\pi(s_2) = 0.2$, and $\pi(s_3) = 0.5$, and riskless rate of return of 8%:

(i) What is the value of an asset that pays $30 in state 1, $40 in state 2 and - $20 in state 3?

A share of common stock will pay $100 in state 1, $117 in state 2, and $205 in state 3. (ii) What is the value of the stock today?
(iii) What is the value today of a call option on that stock with an exercise price of $105? (iv) $195?

**Ans.** (i) The value of the asset is the expected cash flow using the risk neutral probabilities discounted at 8% or

\[
(30(.3) + 40(.2) - 20(.5))/1.08 = (7)/1.08 = 6.48.
\]

(ii) The stock is worth \( (100(.3) + 117(.2) + 205(.5))/1.08 = 155.9/1.08 = 144.35. \)

(iii) The option has a payoff of zero in state 1, \( 117 - 105 = 12 \) in state 2, and \( 205 - 105 = 100 \) in state 3. Thus it is worth \( (0(.3) + 12(.2) + 100(.5))/1.08 = 52.4/1.08 = 48.52. \) (iv) If the exercise price is 195, the option payoff is zero in states 1 and 2 and it is \( 205 - 195 = 10 \) in state 3. It is worth \( 10(.5)/ 1.08 = 4.63. \)
Oil Extraction Problem

Consider a firm that has the right to extract and sell oil from a given well site. The cash flows from this project vary directly with the price of oil. The oil price dynamics are as pictured below:

```
S_0  S_1  S_2
  
20  36  64.8
  |
20  12  21.6
  |
12  7.2
```

$S_0$ is the current spot price of oil per barrel, $S_1$ is the spot price of oil at date 1, and $S_2$ is the spot price of oil at date 2. Cash flow from oil extraction is assumed to be a fixed perpetuity of 1,000,000 barrels per period times the price per barrel of oil. The market value dynamics of oil extraction is pictured below. $V_0$ is the market value of beginning oil extraction at date 0. $V_1(V_2)$ is the market value of beginning extraction at date 1 (2).
Each move in the value tree corresponds to the same move in the price tree. For example, the move from $V_0 = 100$ to $V_1 = 180$ occurs if and only if the price of oil moves from $S_0 = 20$ to $S_1 = 36$.

The market values in the value tree are consistent with cash flows if the required rate of return for oil extraction is 20% and the probability of an up move in oil prices is 0.5 at every node in the tree. We assume this. Check that at each node $V_t = [0.5V_{t+1}^+ + 0.5V_{t+1}^-](1.20)^{-1}$, where a plus superscript indicates the up branch (following from the $V_t$ node) and a minus superscript indicates the down branch. This notation will be used throughout.

Also verify at each node that the up branch is 1.80 times the value at the node and the down branch is .60 times the value at the node. Thus $r_u = 80\%$ and $r_d = -40\%$ everywhere in the tree. Assume the risk free return is $r_f = 8\%$ per period.
Let $I_t$ denote the investment required to begin extraction at date $t$ and assume for simplicity that this is the only cost of extraction that is not proportional to the price of oil. Do the following exercises.

Exercises

Suppose $I_0 = I_1 = I_2 = 104$ million dollars.

1. Calculate the value as of $t = 0$ of the investment policy “do not invest at $t = 0$, invest at $t = 1$ if oil prices go up, do not invest at $t = 1$ if oil prices go down.” Use risk neutral valuation arguments.

2. Calculate the value as of $t = 0$ of waiting until $t = 2$ and optimally exercising the option to invest at $t = 2$.

3. Using the answers from 1 and 2, determine the optimal exercise of the option to invest at $t = 1$.

4. Using the answers in 3, determine the value at $t = 0$ of the oil extraction project.
Suppose that $I_0 = 104$, $I_1 = 112.32$, and $I_2 = 121.31$ (all in millions of dollars).

5. Do 2, 3, and 4 again.

Suppose that $I_0 = 104$, $I_1 = 112.32$, and $I_2 = 150$.

6. Do 2, 3, and 4 again.

Suppose that $I_0 = 90$, $I_1 = I_2 = 104$.

7. The NPV(extract now) = $-90 + 100 = 10$. Should we extract now?
Investment Incentives and Risk Management

One example of an agency problem between shareholders and creditors is the *underinvestment problem*. This arises because the shareholders of a financially distressed or financially troubled firm may not undertake some positive NPV investment alternatives. While a project might have a positive NPV, shareholders will only have an incentive to invest funds in the project and undertake it if their share of the returns from the project justify the investment. In the event that there is some debt outstanding, and the firm is financially distressed, existing creditors will have a claim on a portion of the cash flows of any new project. In the event that the creditors' share of these cash flows is relatively large, shareholders will not find it profitable to invest in the project. The creditors, on the other hand, will
prefer the shareholders to undertake the project and increase the value of their claims. The divergent incentives of the shareholders and creditors with respect to the investment decision constitute an agency problem.

As an example, suppose we are in a setting with two dates and three states of nature $S = \{s_1, s_2, s_3\}$. The risk neutral probabilities of these states are $\pi(s_1) = 0.60$, $\pi(s_2) = 0.30$, and $\pi(s_3) = 0.10$. For simplicity we assume that the risk free rate of return is zero. This assumption just facilitates calculating market values (no rounding required) and in no way changes the qualitative nature of the conclusions here. Under this assumption, the risk neutral probability of a state is the price today of a dollar deliverable in that state. In general, the higher the gross domestic income in a state, the lower the price of a dollar deliverable in that state. So state $s_1$ has the lowest gross
domestic income, state $s_2$ has the next lowest gross domestic income, etc.

Consider the problem of Shaky Steel, Inc., a small steel manufacturer whose cash flows depend upon the price of steel. The price of steel is typically highest in states of the economy when gross domestic income is high and lowest when gross domestic income is low. Shaky Steel’s assets in place will generate end of period cash flows as follows:

<table>
<thead>
<tr>
<th>State</th>
<th>RiskNeutral Probability $\pi(s)$</th>
<th>Cash Flow $X(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>0.60</td>
<td>20,000</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0.30</td>
<td>100,000</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0.10</td>
<td>400,000</td>
</tr>
</tbody>
</table>

Shaky has debt outstanding with a promised payment of $150,000. Shaky is clearly in financial distress. Calculating market values, we get that the total market
value of Shaky Steel is $V_0 = 20,000(0.6) + 100,000(0.3) + 400,000(0.1) = $82,000$, the market value of Shaky’s debt is $D_0 = 20,000(0.6) + 100,000(0.3) + 150,000(0.1) = $57,000$, and consequently, the market value of Shaky’s equity is $E_0 = V_0 - D_0 = $25,000 (also $= 0.1(250,000)$).

Suppose that Shaky discovers a cost saving renovation of its steel production operations that will generate end of period additional net cash flows as follows:

<table>
<thead>
<tr>
<th>State</th>
<th>CashFlow</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td>$Y(s)$</td>
</tr>
<tr>
<td>$s_1$</td>
<td>10,000</td>
</tr>
<tr>
<td>$s_2$</td>
<td>10,000</td>
</tr>
<tr>
<td>$s_3$</td>
<td>30,000</td>
</tr>
</tbody>
</table>

If Shaky undertakes this renovation, then the firm's end of period cash flows would be as follows:
<table>
<thead>
<tr>
<th>State</th>
<th>Probability</th>
<th>CashFlow inPlace</th>
<th>CashFlow Investment</th>
<th>Total Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>π(s)</td>
<td>X(s)</td>
<td>Y(s)</td>
<td>X(s) + Y(s)</td>
</tr>
<tr>
<td>s₁</td>
<td>0.6</td>
<td>20,000</td>
<td>10,000</td>
<td>30,000</td>
</tr>
<tr>
<td>s₂</td>
<td>0.3</td>
<td>100,000</td>
<td>10,000</td>
<td>110,000</td>
</tr>
<tr>
<td>s₃</td>
<td>0.1</td>
<td>400,000</td>
<td>30,000</td>
<td>430,000</td>
</tr>
</tbody>
</table>

If Shaky undertakes the renovation, the total market value of Shaky becomes \( V_0' = 30,000(0.6) + 110,000(0.3) + 430,000(0.1) = $94,000 \), the market value of its debt becomes \( D_0' = 30,000(0.6) + 110,000(0.3) + 150,000(0.1) = $66,000 \), and the market value of its equity becomes \( E_0' = V_0' - D_0' = $28,000 \). Thus if Shaky decides to undertake the renovation, the value of the firm will go up by the value of the new investment \( V_n = 10,000(0.9) + 30,000(0.1) = $12,000 \). Creditors' claims will increase in value by $9,000 \( (= D_0' - D_0) \) while shareholders' equity
will increase in value by $3,000 (\(= E'_0 - E_0 \)). Everyone appears to be better off if the renovation is undertaken.

But we have forgotten who pays for this new investment. Suppose for concreteness that the renovation has an initial cost of $5,000. Then \(\text{NPV} = V_n - 5,000 = 7,000 > 0\) and the investment appears worthwhile. Will the shareholders of Shaky supply the needed $5,000 in equity capital? If they do they would be giving up stock plus cash worth $25,000 + $5,000 = $30,000 for stock worth $28,000. It is clearly not in their interest to do this. The problem here is that even though the investment will add value to the firm of $7,000, such a good investment improves creditors' claim by $9,000 leaving a net loss for shareholders of $2,000. From their perspective they would be better off foregoing this positive NPV renovation.

Exercise: (i) Would shareholders in this case borrow the needed $5,000 to undertake the new investment and
thereby avoid foregoing the positive NPV? (ii) Suppose Shaky has 1,000 shares of common stock outstanding before the new project is undertaken. Would the owners of these shares vote to sell common stock to finance the renovation?

Consider now what would happen to shareholders incentives to undertake the renovation if Shaky had managed its steel price risk. Suppose, in particular, that Shaky can customize a swap arrangement wherein it has the following cash flows (these swap cash flows are incremental to all other cash flows):

<table>
<thead>
<tr>
<th>State</th>
<th>SwapFlow</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>(W(s))</td>
</tr>
<tr>
<td>(s_1)</td>
<td>30,000</td>
</tr>
<tr>
<td>(s_2)</td>
<td>-75,000</td>
</tr>
<tr>
<td>(s_3)</td>
<td>45,000</td>
</tr>
</tbody>
</table>

With this swap arrangement, the valuation picture looks as follows:
<table>
<thead>
<tr>
<th>State</th>
<th>Probability</th>
<th>CashFlow with Renovation +</th>
<th>CashFlow from Swap =</th>
<th>Total Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>$\pi(s)$</td>
<td>$X(s) + Y(s)$</td>
<td>$W(s)$</td>
<td>$X(s) + Y(s) + W(s)$</td>
</tr>
<tr>
<td>$s_1$</td>
<td>0.6</td>
<td>30,000</td>
<td>30,000</td>
<td>60,000</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0.3</td>
<td>110,000</td>
<td>-75,000</td>
<td>35,000</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0.1</td>
<td>430,000</td>
<td>45,000</td>
<td>475,000</td>
</tr>
</tbody>
</table>

Calculating market values (you do the calculations), we get that the market value of Shaky is still $V_0'' = $94,000. The reason for this is that the value of the swap cash flows is zero as it should be (check this yourself). But now the market value of Shaky’s debt is $D_0'' = $61,500, leaving the equity worth $E_0'' = 94,000 - 61,500 = $32,500. This indicates that Shaky’s shareholders would be better off undertaking the renovation with the customized swap arrangement in place. They would be trading old equity plus newly contributed equity worth $25,000 + $5,000 =
$30,000 for equity worth $32,500, thereby gaining $2,500.

The key here is that the risk management strategy removed the force of the debt overhang by removing the ability of the debt to garner all of the NPV of the project plus some of shareholders wealth. Because the value of the swapped cash flows is zero, this risk management strategy is a costless way to restore shareholders incentive to add value to the firm.
Unfortunately, this example points up another agency problem between shareholders and creditors. This problem with debt financing is called the risk shifting or asset substitution problem. Once debt is issued, it is in shareholders interest to change the firm’s cash flows in ways that increase the risk of these cash flows since such increases in risk cause creditors claims to be devalued. Such changes are referred to as asset substitutions, and their character gives rise to the term risk shifting.

The swap cash flow in the case of Shaky Steel, Inc., offers a perfect example. Consider again Shaky with just assets in place that generate the cash flow $X(s)$. If Shaky undertakes the customized swap arrangement, its cash flows become $X(s) + W(s)$. This addition of $W(s)$ causes an increase of cash in the extreme states $s_1$ and $s_3$, but a decrease in cash flow in the middle state $s_2$. Any alteration of cash flow that moves cash from the middle of the
distribution to the extremes is (almost by definition) an increase in risk of those cash flows.

The result of this increase in risk is a devaluation in Shaky’s debt. To see this, consider Shaky with the swap in place, but without the cost saving renovation of its steel production operations.

<table>
<thead>
<tr>
<th>State</th>
<th>Probability</th>
<th>CashFlow inPlace</th>
<th>Cash Swap</th>
<th>Total Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>π(s)</td>
<td>X(s)</td>
<td>W(s)</td>
<td>X(s) + W(s)</td>
</tr>
<tr>
<td>s₁</td>
<td>0.6</td>
<td>20,000</td>
<td>30,000</td>
<td>50,000</td>
</tr>
<tr>
<td>s₂</td>
<td>0.3</td>
<td>100,000</td>
<td>-75,000</td>
<td>25,000</td>
</tr>
<tr>
<td>s₃</td>
<td>0.1</td>
<td>400,000</td>
<td>45,000</td>
<td>445,000</td>
</tr>
</tbody>
</table>

Since the value of the swap cash flow is zero, the total market value of shaky is still \( V_0 = \$82,000 \), as you should verify by direct computation. Notice, however that the value of Shaky’s debt with the swap in place is \( D_0'''' = 50,000(0.6) + 25,000(0.3) + 150,000(0.1) = \$52,500 \). The
risk shift implicit in the swap cash flows has caused Shaky’s debt to go from \( D_0 = \$57,000 \) to \( D_0'' = \$52,500 \), a loss of \( \$4,500 \). Since the total value of shaky has not changed because the value of the swap cash flow is zero, the value of Shaky’s equity will have to increase by \( \$4,500 \). By direct calculation, \( E_0''' = 295,000(0.1) = \$29,500 = E_0 + 4,500 \).

The sad result is that because of this incentive to transfer wealth from creditors to shareholders by shifting risk, the ability of the swap to restore shareholders incentives to undertake the renovation has been destroyed. For once the swap is in place, shareholders face exactly the same under investment problem they had before the swap. Their debt is worth \( D_0''' = \$52,500 \) and is risky. With the renovation, the debt will be worth \( D_0'' = \$61,500 \), an increase of \( \$9,000 \), which is again the \$7,000
of positive NPV from the renovation project, plus $2,000 more of shareholders’ wealth.

Shareholders have no incentive to undertake this renovation. In essence, shareholders would be swapping equity worth $E_0^{***} = 29,500$ and cash of $5,000$, total wealth worth $34,500$, for equity worth $E_0^{**} = 32,500$. It is clearly not in their interest to do this.

Thus swaps that are increases in risk have these perverse incentives. What about swaps that are decreases in risk? Ones that move cash flow from the extremes of the distribution to the middle? Can these kinds of swaps produce the right incentives? The answer is again no. To see this in Shaky’s case, consider the following customized swap arrangement.
State  \begin{align*}
  s & \quad \hat{W}(s) \\
  s_1 & \quad -20,000 \\
  s_2 & \quad 55,000 \\
  s_3 & \quad -45,000
\end{align*}

You should verify by direct calculation that the value of this swap is zero. With this swap in place, Shaky’s cash flows are as follows:

<table>
<thead>
<tr>
<th>State</th>
<th>Risk</th>
<th>CashFlow</th>
<th>Swap</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>Neutral</td>
<td>(\pi(s))</td>
<td>(X(s))</td>
<td>(\hat{W}(s))</td>
</tr>
<tr>
<td>s_1</td>
<td>0.6</td>
<td>20,000</td>
<td>-20,000</td>
<td>0</td>
</tr>
<tr>
<td>s_2</td>
<td>0.3</td>
<td>100,000</td>
<td>55,000</td>
<td>155,000</td>
</tr>
<tr>
<td>s_3</td>
<td>0.1</td>
<td>400,000</td>
<td>-45,000</td>
<td>355,000</td>
</tr>
</tbody>
</table>

You should also verify that \(D_0^{****} = 150,000(0.4) = 60,000\) and \(E_0^{****} = 82,000 - 60,000 = 22,000\). Thus relative to Shaky with only assets in place, the value of the
debt goes up by $3,000 and the value of the equity goes down by $3,000 with the new swap in place.

While it is clear that the swap alone is not in shareholders interest, is the same true of the swap plus the renovation? After arranging the swap shareholders would consider the renovation. In that case, they would be exchanging equity worth $22,000 plus cash of $5,000, total wealth of $27,000, for equity worth $28,000 (verify that Shaky with \( X(s) + \hat{W}(s) + Y(s) \), with the new swap and the renovation, has the same total value as with the other swap and renovation, $94,000, but the value of Shaky’s debt is now $66,000 and hence the value of the equity is $28,000).

So after the swap the renovation would be undertaken. Would shareholders arrange the swap to begin with, knowing that the equity would decline by $3,000? The answer is no. In essence, they would be exchanging
equity worth $25,000 plus cash of $5,000, for equity worth $28,000. Just as before, this is not in shareholders’ interest.

The effect of the new swap is to make creditors’ claim less risky and hence it is revalued upward, costing shareholders $3,000. While creditors’ after the swap then only garner $6,000 of the positive NPV of the renovation, leaving $1,000 for shareholders, the net result is again an increase in creditors’ claims of $9,000, which is the $7,000 NPV of the renovation and $2,000 more of shareholders’ wealth.