Fi8000
Valuation of
Financial Assets

Spring Semester 2010
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Assistant Professor of Finance

Today

○ Syllabus
○ Course overview
○ Lecture: Time Value of Money
  - Brealey, Myers and Allen, Principles of Corporate Finance, McGraw Hill, 8th edition – Chapter 3
  - Ross, Westerfield and Jaffe, Corporate Finance, McGraw Hill, 8th edition – Chapter 5

Syllabus

○ Expectations
  ○ Be here
  ○ Ask questions
  ○ Work practice problems
○ Required skills
  ○ Spreadsheet (Excel) and Internet
○ Required materials
  ○ Investments, Bodie, Kane and Marcus, 8th edition

Syllabus

○ Grading
  ○ 2 Midterm Exams @ 25% each
  ○ Final exam @ 40%
  ○ Stock-Trak assignment @ 10%
○ Make-up Policy
  ○ No make-up quizzes or exams
  ○ Everyone must take the final exam

Syllabus

○ Grading – Typical Department Policy
  ○ No more than 35% A (A or A-)
  ○ Majority (approximately 50%) B (B+, B or B-)
  ○ Lagging performance earns a C+ or lower
○ Administrative – Withdrawal with a WF
  ○ Beyond 3 absences from class
  ○ Withdrawal after the semester midpoint
  ○ Withdrawal while doing failing work

Syllabus

○ Materials
  ○ (Financial) Calculator – bring every day
  ○ Text – leave it where you read it
  ○ Lecture notes – on the web page
○ Office Hours
  ○ Drop-in, phone, e-mail and by appointment
Approaches to Valuation

- Discounted cash flows
  The value of an asset is related to the stream of expected cash flows that it generates, and should reflect compensation for time and risk.

- Arbitrage pricing
  When two assets have exactly the same stream of cash flows (magnitude, date, state) their prices should be identical.

Discounted Cash Flows (DCF) Valuation

The Idea

The value of an asset is the present value of its expected cash flows.

Inputs for DCF valuation

The magnitude, timing and risk level of the expected CFs

Assumptions Underlying DCF Valuation

- **Magnitude**: investors prefer to have more rather than less.
- **Timing**: investors prefer a dollar today rather than a dollar some time in the future.
- **Risk**: investors would rather get a certain CF of $1 than get a lottery ticket with an expected (average) CF of $1.

The Mechanics of Time Value

**Compounding**

Converts present cash flows into future cash flows.

**Discounting**

Converts future cash flows into present cash flows.

The Additivity Principal

Cash flows at different points in time cannot be compared or aggregated. All cash flows have to be brought to the same point in time before comparisons or aggregations can be made.

Notation

- $ PV $ = Present Value
- $ FV $ = Future Value
- $ CF_t $ = Cash Flow on date $ t$
- $ t $ is the time (date) index ($ t = 1, 2, \ldots, T $)
- $ k $ = risk-adjusted discount rate
- $ r_f $ = risk-free discount rate
- $ g $ = growth rate

Compounding

$ FV = PV \cdot (1 + k)^T $
Discounting

$$PV = \frac{100}{1.05^0} \approx 100$$

$$PV = \frac{100}{1.05^1} \approx 95.24$$

$$PV = \frac{100}{1.05^2} \approx 90.70$$

Examples

1. How much will you pay today for a project that is expected to pay a dividend of $500,000 three years from now, if the appropriate (risk-adjusted) annual discount rate for this project is 10%?

2. What is the value of a project that is expected to pay $150,000 one year from now and $500,000 three years from now, if the appropriate (risk-adjusted) annual discount rate for this project is 10%?

Solutions

1. $$PV = \frac{500,000}{(1+0.1)^3} = 375,657.40$$

2. $$PV = \frac{150,000}{(1+0.1)^1} + \frac{500,000}{(1+0.1)^3} = 136,363.64 + 375,657.40 = 512,021.04$$

Cash Flow Streams – special cases

A growing perpetuity:

$$PV = \frac{CF_1}{(1+k)} + \frac{CF_1(1+g)}{(1+k)^2} + \frac{CF_1(1+g)^2}{(1+k)^3} + ... = \sum_{t=0}^{\infty} \frac{CF_1(1+g)^t}{(1+k)^t} = \frac{CF_1}{1-g} \quad (if \quad k > g)$$

Example

Southwestern Bell is expected to pay a dividend of $2.89 per share one year from now. Its earnings and dividends had grown at 6% a year in the last 5 years and they are expected to grow at the same rate in the long term. The rate of return required by investors on stocks of equivalent risk was 12.23%.

What should be the price / value of the stock today?
Solution

\( CF_1 = $2.89 \)

Expected growth rate \( g = 6\% = 0.06 \)

Discount rate \( k = 12.23\% = 0.1223 \)

\[
PV = \frac{CF_1}{(1 + k) - 0.06} = $46.39
\]

Example - continued

In fact, the stock is actually trading at $70 per share. This price could be justified by using a higher expected growth rate.

\[
\frac{70}{2.89} \frac{1}{1.1223 - g} = 8.1\%
\]

Cash Flow Streams – special cases

A growing annuity:

\[
PV = \sum_{t=1}^{T} \frac{CF_1(1+g)^{t-1}}{(1+k)^t} = \frac{CF_1}{(1+k)^2} + \frac{CF_1(1+g)}{(1+k)^2} + \ldots + \frac{CF_1(1+g)^{T-1}}{(1+k)^T}
\]

Example

Suppose you are trying to borrow $200,000 to buy a house on a conventional 30-year mortgage with monthly payments.

The monthly interest rate on this loan is 0.70\%. What is the monthly payment on the loan?

Solution

\( PV = $200,000 \)

\( T = 30 \times 12 = 360 \)

\( k = 0.70\% = 0.007 \)

\( g = 0 \)

\( \frac{200,000}{0.007} \left[ 1 - \left( \frac{1}{1 + 0.007} \right)^{360} \right] = $1,523.7 \)

The Frequency of Compounding

The frequency of compounding affects both the future and present values of cash flows.

The quoted annual interest rate may be compounded more frequently than once a year. This will affect the effective annual interest rate which is determined by the frequency of compounding.
Example
The annual interest rate on a loan with monthly payments is 8.4%. The quoted annual rate is 8.4%, compounded monthly.

What is the effective annual rate on the loan?
What is the effective quarterly rate on the loan?
What is the effective weekly rate on the loan?

Quoted to Effective
The quoted annual rate is 8.4%, compounded monthly.
This rate is NOT effective since the frequency of compounding is monthly, always start by finding the effective monthly rate. There are m=12 months in one year, so

\[
r_{\text{effective, monthly}} = \frac{r_{\text{quoted, annual}, m}}{m} = \frac{8.4\%}{12} = 0.70\%
\]

Effective to Effective 1
The effective monthly rate is 0.70%, what is the effective annual rate? Effective quarterly rate?
Since we know the effective monthly rate and there are 12 months in one year, the effective annual rate is

\[
r_{\text{effective, annual}} = \left(1 + r_{\text{effective, monthly}}\right)^{12} = \left[1 + 0.007\right]^{12} = 1.0873 = 8.73\%
\]

Effective to Effective 2
The effective monthly rate is 0.70%, what is the effective annual rate? Effective quarterly rate?
Since we know the effective monthly rate and there are 3 months in one quarter, the effective quarterly rate is

\[
r_{\text{effective, quarterly}} = \left(1 + r_{\text{effective, monthly}}\right)^{3} = \left[1 + 0.007\right]^{3} = 1.02115 = 2.115\%
\]

Effective to Effective 3
The effective monthly rate is 0.70%, what is the effective weekly rate?
Since we know the effective monthly rate and there are 4 weeks in one month (0.25 months in one week), the effective weekly rate is

\[
r_{\text{effective, weekly}} = \left[1 + r_{\text{effective, monthly}}\right]^{\frac{1}{4}} = \left[1 + 0.007\right]^{\frac{1}{4}} = 1.0017 = 0.17\%
\]

Example Cont.
The annual interest rate on a loan with monthly payments is 8.4%. The quoted annual rate is 8.4%, compounded monthly.

What is the effective annual rate on the loan?

\[
1 + r_{\text{effective, annual}} = \left[1 + \frac{r_{\text{quoted, annual}}}{m}\right]^{m} = \left[1 + \frac{0.084}{12}\right]^{12} = 1.0873 = 8.73\%
\]
Continuously Compounded Rates

\[ 1 + r_{\text{effective}, \text{annual}} = \left( 1 + \frac{r_{\text{quoted}, \text{annual}}}{m} \right)^m \]

As \( m \) (the frequency of compounding) goes to infinity (\( m \rightarrow \infty \)) we get

\[ 1 + r_{\text{effective}, \text{annual}} = e^{r_{\text{quoted}, \text{annual}}} \]

The Frequency of Compounding

<table>
<thead>
<tr>
<th>Frequency of Compounding</th>
<th>Quoted Rate</th>
<th>m</th>
<th>Effective Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>annual</td>
<td>10%</td>
<td>1</td>
<td>10.00%</td>
</tr>
<tr>
<td>semi-annual</td>
<td>10%</td>
<td>2</td>
<td>10.25%</td>
</tr>
<tr>
<td>monthly</td>
<td>10%</td>
<td>12</td>
<td>10.47%</td>
</tr>
<tr>
<td>weekly</td>
<td>10%</td>
<td>52</td>
<td>10.51%</td>
</tr>
<tr>
<td>continuous</td>
<td>10%</td>
<td>( \infty )</td>
<td>10.52%</td>
</tr>
</tbody>
</table>

Quote to Effective: Examples

The quoted annual rate is 12%, compounded semiannually. What is the effective semi-annual rate? (6%)

The quoted monthly rate is 3%, compounded weekly. What is the effective weekly rate? (0.75%)

The quoted quarterly rate is 4%, compounded monthly. What is the effective monthly rate? (1.33%)

Effective to Effective: Examples

The effective monthly rate is 1.2%. What is the effective semi-annual rate? (7.42%)

The effective quarterly rate is 3%. What is the effective annual rate? (12.55%)

The effective annual rate is 12%. What is the effective monthly rate? (0.945%)

Examples

The quoted annual rate of return is 10%, compounded semiannually. Calculate the following rates:

a. Effective rate for 6 months (5%)
b. Effective rate for 2 years (21.55%)
c. Effective rate for 1 year (10.25%)
d. Effective rate for 18 months (15.76%)
e. Effective rate for 2 months (1.64%)