Precoloring Extensions using Hall’s Condition

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In 1990 Hilton and Johnson introduced Hall’s condition, a generalization of Hall’s Marriage Theorem applied to list assignments of the vertex set of a graph. A list assignment $L$ to a graph $G$ is called Hall if $(G, L)$ satisfy Hall’s condition, which is necessary (but not always sufficient) for $G$ to admit a proper $L$-coloring. A graph $G$ is Hall $m$-completable if every proper $m$-precoloring of $G$, whose corresponding list assignment is Hall, can be extended to a proper $m$-coloring of $G$. In 2011, Bobga et al. asked if a graph $G$ is always Hall $\Delta(G)$-completable, thereby posing a possible precoloring extension version of Brooks’ theorem. We give a straightforward and short proof that answers this question in the affirmative as well as discuss some related results and open questions. Additionally, for several graph families, we will examine the full spectrum of values for which a graph is Hall $m$-completable. This yields some surprising results, such as graphs which are both Hall $m$-completable and Hall $(m + k)$-completable, for infinitely many $k > 1$, but are not Hall $(m + 1)$-completable. This is joint work with Sarah Holliday and Jennifer Vandenbussche.